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The Natural Redistribution of Sunlight and Skylight Due to the Atmospheric Turbidity of Cloudless Skies

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ABSTRACT
The transmittance and absorption of sunlight as well as the scattering properties of the atmosphere can be characterized by the luminous turbidity factor, which also determines the redistribution of sunlight and skylight at the ground level. The International Organization for Standardization (ISO)/CIE cloudless sky types defining the quasi-homogeneous luminance sky patterns enable calculation of either the relative distribution of luminance on the sky vault or the redistribution of absolute sunlight and skylight illuminance levels on the ground. Applying appropriate formulae, this article shows the consequences of turbidity changes on such redistribution in the case of rising solar altitudes; for example, when the ISO/CIE clear sky type 12 occurs during a summer day.

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1. Introduction

Under clear cloudless sky conditions when parallel sunbeams with simultaneous diffuse skylight from the whole sky vault are present at ground level, their mutual participation is determined by the momentary solar altitude and the overall turbidity of the atmosphere. All sunlight reaching the outer border of the Earth's atmosphere is defined by the luminous flux equal to the average luminous solar constant E_{vo} = 133.8 klux [Darula and others 2005]. Due to the solar altitude at a certain locality on Earth, the reference fictitious horizontal extraterrestrial illuminance $E_{vo,h}$ is thus determined as

$$E_{vo,h} = E_{vo} \sin \gamma_s = 133.8 \sin \gamma_s \ (\text{klx}), \qquad (1)$$

where γ_s is the solar altitude.

E_{vo}, the luminous solar constant, is valid for the average yearly distance of the Earth from the Sun (149.59 Mkm) on April 3 and October 5 with fluent variations of ±3.3% to a winter maximum on January 3 and a summer minimum on July 4. If necessary, a correction for any date within a year can be made, but the average 133.8 klx will be used for technical applications in this article.

When daylight science and practical daylight design are trying to change their criteria from relative to absolute illuminance/luminance levels, the extraterrestrial influx into exterior and interior spaces is to be based on the available solar and sky resources [Reinhart 2013; Tregenza and Wilson 2011]. Therefore, it is important to study their natural redistribution while explaining multiple influences of the atmosphere.

2. Direct sunlight illuminance at the horizontal ground level

The transmittance of the atmosphere or its absorption was studied in the 18th century and formulated as Bouguer's or Lambert's law [IESNA 2000], now used as pollution content after the Linke turbidity factor T_L in the whole sun radiation spectrum. Using the luminous alternative T_v, the attenuation of sunlight is expressed by the formula

$$E_{vs} = E_{vo,h} \exp(-a_v \, m \, T_v) \ (\text{klx}), \qquad (2)$$

where E_{vs} is the solar illuminance at the horizontal ground level; $E_{vo,h}$ is the horizontal extraterrestrial fictitious illuminance after (1); a_v is the luminous extinction coefficient of the absolutely clear and clean atmosphere, defined by Clear [1982] and Navvab and others [1984], which is approximately

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a_v = 0.1 but also depends on the relative air mass m; in addition [Clear 1982],

$$a_v = \frac{1}{9.9 + 0.043\,m}, \tag{3}$$

where m is the relative air mass and depends on the solar altitude originally proposed by Bouguer [1729] as

$$m = \frac{1}{\sin \gamma_s}, \tag{4}$$

but now a more precise formula, especially for low altitudes close to the horizon due to the reduced thickness of air mass around the globe, is recommended after Kasten and Young's [1989] approximation formula :

$$m = \frac{1}{\sin \gamma_s + 0.50572\left(\gamma_s + 6.07995°\right)^{-1.6364}}. \tag{5}$$

However, for solar altitudes over 20°. values m after (4) and (5) are the same.

When either E_{vs} or the global horizontal illuminance E_{vg} and diffuse E_{vd} are measured, then using (2), [Linke and Boda 1922],

$$T_L = \frac{\ln E_{eo} - \ln E_{eo}\ \exp(-a_r\, m\, T_L)}{a_r\, m} = \frac{\ln E_{eo,h} - \ln\left(E_{eg} - E_{ed}\right)}{a_r\, m} \tag{6}$$

and in luminous variables is

$$T_v = \frac{\ln E_{vo,h} - E_{vs}}{a_v\, m} = \frac{\ln E_{vo,h} - \ln\left(E_{vg} - E_{vd}\right)}{a_v\, m}. \tag{7}$$

When neither momentary or regular irradiance $E_{eo,h}$, E_{eg}, and E_{ed} are measured and illuminance E_{vs} or E_{vg} and E_{vd} are available in a certain locality, then average T_L and T_v have to be assumed in computer programs.

Under cloudless conditions, the overall filtration of the atmosphere can be determined from the atmospheric turbidity containing molecular as well as pollution influences expressed by the luminous turbidity factor T_v [Kittler and others 1992]. There are two possibilities to roughly estimate the T_v value. After measurement averages, the lowest turbidity in mountainous regions and countryside settlements is approximately T_v = 2, whereas in towns T_v = 4 is often approached. In polluted industrial regions, a considerable reduction in sunlight is characterized by T_v = 5–6. However, some yearly variations in T_L were found (such as those recommended by Aydinli and Krochmann [1983]), as summarized in Table 1, which are approximately similar to monthly T_v changes.

Low luminous turbidity T_v occurs only during cloudless periods or days when no cloud cover hinders the direct penetration of sunbeams toward the ground. Such sunny periods are approximated at meteorological stations worldwide using the Campbell-Stokes glass sphere heliograph, which produces sunburned traces on hard paper strips that indicate sunshine duration or, alternatively, are calculated from measured diffuse and global irradiances. Then, the relative sunshine durations can be determined as ratios of maximally possible astronomical sunshine duration during a day, week, month, or year. However, such unnaturally frequent changes in sunshine periods that are weaker or stronger in sunlight intensities render the value of the turbidity factor uncertain. Therefore, the true momentary value of turbidity should be determined from locally measured direct sunlight illuminance.

3. Scattered diffuse skylight from the sky vault luminance received at the ground level

Once the measurement is available, a good approximation of the overall atmospheric transmittance and pollution turbidity filter is achieved and the

Table 1. Monthly averages of the turbidity values T_L in the whole solar spectrum for the moderate climate of Europe.

Month	January	February	March	April	May	June	July	August	September	October	November	December	Yearly
Mountains	1.5	1.6	1.8	1.9	2.0	2.3	2.3	2.3	2.1	1.8	1.6	1.5	1.90
Country	2.1	2.2	2.5	2.9	3.2	3.4	3.5	3.3	2.9	2.6	2.3	2.2	2.75
Towns	3.1	3.2	3.5	4.0	4.2	4.3	4.4	4.3	4.0	3.6	3.3	3.1	3.75
Industrial	4.1	4.3	4.7	5.3	5.5	5.7	5.8	5.7	5.3	4.9	4.5	4.2	5.00

cloudless homogeneous sky is then matched with the effect of direct parallel sunbeams reaching the ground level. If the T_v value is close to one, it means that the momentary state of the atmosphere is relatively clean and unpolluted, whereas if T_v = 4, it can be imagined that the current atmosphere consists of four layers of an absolutely clear state. Thus, a considerable reduction in direct sunlight after (2) is determined, whereas a certain increase of skylight is occurring in accordance with the different International Organization for Standardization [ISO 2004]/CIE [2003] cloudless sky types. The resulting zenith luminance L_{vZ} and luminance pattern as well as their horizontal diffuse illuminance at the ground level E_{vd} follow the complex formulae [Darula and Kittler 2005]:

$$L_{vZ} = T_v\left[0.7\,X + A1\sin\gamma_s + 0.04\right] + 0.7\,X + A2\sin\gamma_s,\ \left[\mathrm{kcd/m^2}\right] \quad (8)$$

$$E_{vd} = \frac{L_{vZ}\,E_{vo,h}}{B\,X + E\sin\gamma_s}\ [\mathrm{klx}], \quad (9)$$

where $X = \frac{(\sin\gamma_s)^C}{(\cos\gamma_s)^D}$ and $A1$, $A2$, B, C, D, E are parameters for all ISO/CIE sky types 7–15 with sun presence given in Kittler and others [2012].

The resulting horizontal global (sun and skylight) illuminance $E_{v,g}$ is

$$E_{vg} = E_{vo}\sin\gamma_s\left[\exp(-a_v\ m\,T_v) + \frac{L_{vZ}}{B\,X + E\sin\gamma_s}\right](\mathrm{klx}), \quad (10)$$

where recommended parameter a_v is to be calculated either after (3) or as an approximate constant value 0.1 and m either for higher solar altitudes after (4) or after (5).

Both relative sun and sky components within the brackets of (10) express their dependence on solar altitude and turbidity conditions, and their intensities are given by the momentary fictitious horizontal extraterrestrial level.

4. Example of the sunlight and skylight redistribution for the iso/cie clear sky types

Although (10) is valid for ISO/CIE cloudless sky types 11–15, the extreme redistribution following higher solar altitudes in the usual range of luminous turbidity occurs in the cloudless and clear (blue) luminance patterns when the ISO/CIE sky type 12 is present. Table 2 summarizes sun and sky components for various solar altitudes 20–60° in 10° steps under luminous turbidity T_v = 2.5, 4.5, and 6.5. For simplicity, we assumed a_v = 0.1 and $m = \frac{1}{\sin\gamma_s}$, and the constant recommended parameters for ISO/CIE sky type 12 were applied; that is, $A1$ = 1.036, $A2$ = 0.71, B = 23, C = 4.43, D = 0.74, and E = 18.52.

By applying these components and their multiplication by the horizontal extraterrestrial illuminance, sunlight and skylight illuminance levels as well as the global illuminance level in absolute physical units can be drawn (Figure 1). Similarly, sunlight and skylight illuminance levels corresponding to any solar altitude T_v values in case of different ISO/CIE sky types can be calculated more precisely if necessary. Table 2 shows the average extreme range of usual T_v values with the influence of solar altitude changes during spring and summer days.

Any other sky type with sunshine can be similarly studied determining the relation between sunlight and skylight due to momentary luminous turbidity and solar altitude. For instance, for ISO/CIE sky type 14 the frequently occurring T_v values are in the range 3–5.5 and the following sky type parameters have to be applied: $A1$ = 0.881, $A2$= 0.453, B = 25.54, C = 4.4, D = 0.79, and E = 14.56.

Equations (8)–(10) can be used in a computer program or the ratios $E_{v,s}/E_{vo,h}$ and $E_{v,d}/E_{vo,h}$ can be calculated directly as

$$\frac{E_{vs}}{E_{vo,h}} = \exp(-a_v\ m\,T_v), \quad (11)$$

$$\frac{E_{vd}}{E_{vo,h}} = \left[\frac{L_{vZ}}{B\,X + E\sin\gamma_s}\right], \quad (12)$$

which can serve directly in ratio form to be compared with values in Table 3.

Table 2. Relative sun and sky components indicating the sun and sky illuminance redistribution for ISO/CIE sky type 12.

Solar altitudes	Sun	Sky	Sun	Sky	Sun	Sky
T_v	2.5	2.5	4.5	4.5	6.5	6.5
20°	0.4814	0.1912	0.2683	0.3137	0.1495	0.4362
30°	0.6065	0.1796	0.4065	0.2934	0.2725	0.4071
40°	0.6778	0.1666	0.4965	0.2708	0.3637	0.3751
50°	0.7216	0.1304	0.5559	0.2475	0.4282	0.3419
60°	0.7493	0.1396	0.6376	0.2243	0.5220	0.3091

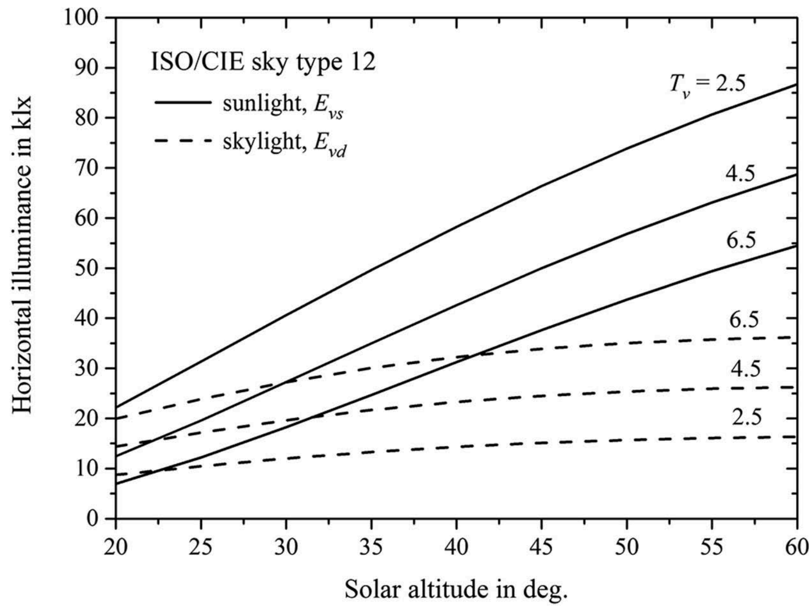

Figure 1. Absolute illuminances assuming luminous solar constant 133.8 klx and typical luminous turbidity under a very clear cloudless sky.

Table 3. Relative sun and sky components indicating the sun and sky illuminance redistribution for ISO/CIE sky type 14.

Solar altitudes	Sun	Sky	Global	Sun	Sky	Global	Sun	Sky	Global
T_v	3.0	3.0	3.0	4.5	4.5	4.5	5.5	5.5	5.5
20°	0.416	0.231	0.647	0.268	0.331	0.599	0.200	0.397	0.597
30°	0.549	0.210	0.759	0.407	0.300	0.707	0.333	0.360	0.693
40°	0.627	0.188	0.815	0.497	0.267	0.764	0.425	0.319	0.744
50°	0.676	0.166	0.842	0.556	0.235	0.791	0.488	0.281	0.769
60°	0.707	0.149	0.856	0.638	0.209	0.847	0.530	0.249	0.779

Similarly, as for sky type 12, these sun and sky components can be multiplied by particular $E_{vo,h}$ values and thus horizontal ground illuminance can be drawn for ISO/CIE sky type 14 in Figure 2.

If both zenith luminance L_{vZ} (kcd/m^2) and horizontal illluminance E_{vd} (klx) are calculated after (8) and (9) for different luminous turbidity T_v, then their ratio L_{vZ}/E_{vd} can be determined as shown in Table 4; for example, for the ISO/CIE clear sky type 12.

However, after modifying (12), a theoretical value L_{vZ}/E_{vd} for any ISO/CIE sky type for any luminous turbidity can be calculated after

$$\frac{L_{vZ}}{E_{vd}} = \frac{B\,X + E\sin\gamma_s}{E_{vo}\sin\gamma_s} = \frac{BX/\sin\gamma_s + E}{133.8}. \qquad (13)$$

Thus, L_{vZ}/E_{vd} for ISO/CIE sky types 12 and 14 are compared with the *ZL* parameter from the IESNA *Lighting Handbook* [IESNA 2000] in Table 5.

Parameters applied in (13) are as follows: for sky type 12: B = 23, C = 4.43, D = 0.74, E = 18.52 and for sky type 14: B = 25.54, C = 4.4, D = 0.79, E = 14.59.

It is evident that $ZL = L_{vZ}/E_{vd}$ for the clear ISO/CIE sky type 12 as given in Table 5 and valid under any luminous turbidity T_v as documented in Table 4. Thus, parameters B, C, D, and E after the CIE guide [CIE 2014] were also verified for L_{vZ} and E_{vd} computer calculations.

5. Conclusions

In a previous study [Kittler and Darula 2015], multiparametric interrelations were tested using all ISO/CIE sky types including overcast and cloudless clear skies taking into account different solar altitudes and atmospheric turbidity conditions influencing the simultaneously occurring sunlight and skylight horizontal illuminance levels at the ground. In the previous paper, we compared theoretical and measured illuminance levels, normalized to extraterrestrially available data in graphical nomograms for sky type

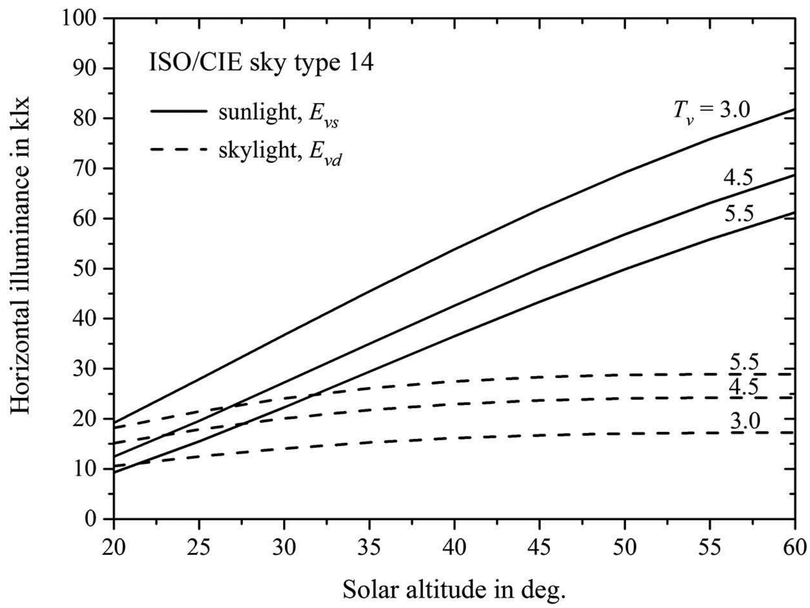

Figure 2. Absolute illuminances assuming luminous solar constant 133.8 klx and typical luminous turbidity under a cloudless, more turbid sky.

Table 4. Absolute zenith luminance and horizontal illuminance with their ratio L_{vZ}/E_{vd} under sky type 12.

γ_s	L_{vZ} (kcd/m^2)			E_{vd} (klx)			L_{vZ}/E_{vd}		
T_v	2.5	4.5	6.5	2.5	4.5	6.5	2.5	4.5	6.5
20°	1.251	2.052	2.853	8.750	14.355	19.960	0.1430	0.1430	0.1430
30°	1.876	3.065	4.253	12.016	19.626	27.236	0.1562	0.1562	0.1562
40°	2.642	4.295	5.948	14.330	23.292	32.254	0.1844	0.1844	0.1844
50°	3.671	5.935	8.198	15.691	25.365	35.038	0.2340	0.2340	0.2340
60°	5.122	8.232	11.343	16.326	26.242	36.158	0.3137	0.3137	0.3137

Table 5. Comparison of L_{vZ}/E_{vd} ratios for sky types 12 and 14 after (13) to IESNA *ZL*.

Solar altitude (°)	L_{vZ}/E_{vd} for sky type 12	IESNA parameter *ZL*	L_{vZ}/E_{vd} for sky type 14
10	0.1388	0.139	0.1093
20	0.1430	0.142	0.1140
30	0.1562	0.156	0.1291
40	0.1844	0.185	0.1613
50	0.2340	0.234	0.2182
60	0.3137	0.314	0.3112

8 with relatively high pollution, represented by luminous turbidity T_v = 5–8 and 8–15, whereas clear sky cloudless sky type 12 represented extreme very clean atmospheres with T_v = 2–3, 2.5–4.5 and for a polluted sky, T_v = 6–7.5. Furthermore, these comparisons were followed by those with daily rising solar altitudes that were compared by changes in levels of direct/sunlight, diffuse/skylight, and global illuminance. Both tests have shown that the theoretical mathematically expressed formulae (8) and (9) or (10) well express real measured zenith luminance as well as illuminance levels at the ground level caused by sunlight and skylight in proportion to corresponding solar altitude and luminous turbidity.

Despite a possible wide range of pollution and turbidity atmospheric content within specific ISO/CIE sky types, the typical L_{vZ}/E_{vd} ratio remains for any particular sky type dependent only on the momentary solar altitude. Thus, the L_{vZ}/E_{vd} ratio can serve as a selection criterion for homogeneous ISO/CIE sky types without regard to any present turbidity conditions; that is, different T_v. However, because both absolute levels of L_{vZ} and E_{vd} with their magnitude and interdependence of sun and sky illuminance participation are affected by the luminous turbidity T_v after (7) and (8), their changes have to be taken into account when determining their absolute influence in real cases. The almost perfect fit of the L_{vZ}/E_{vd} value for the clear sky type 12 with the IESNA sky zenith luminance constants *ZL* for clear sky is astonishing and interesting to note [IESNA

2000]. In this article, the proportions of sunlight and skylight were precisely formulated in ratios as well as in absolute physical units in kilolux related to extraterrestrial fictitious horizontal levels. For computer calculation of global, sun, and sky illuminance distributions under ISO/CIE cloudless skies or their zenith luminance with luminance patterns on the whole sky vault in absolute physical units in kilocandelas per square meter are now determined. General formulae (8) to (10) for any ISO/CIE sky type were developed by applying relatively simple mathematic expressions taking into account extraterrestrially available sunlight flux reaching the Earth's atmospheric border. In accordance with every ISO/CIE standard, the sky luminance distribution due to scattering properties and the turbidity content of the atmosphere create a particular luminance pattern on the sky vault. Due to any window orientation, different sky luminance distributions are present within the solid angle seen from interior surfaces. Thus, the absolute luminance of the sky hemisphere and the resulting illuminance caused by parallel sunbeams and the diffuse sky illuminance under the ever changing solar altitudes and turbidity conditions can be quickly calculated in practical illuminating engineering problems and daylighting design, which is more suitable than graphical nomograms published earlier.

Nomenclature of variables and parameters

$A1$ Parameter defining zenith luminance (see Kittler and others [2012], table 5.2)
$A2$ Parameter defining zenith luminance (see Kittler and others [2012], table 5.2)
a_r Exctinction coefficient of the solar radiation spectrum
a_v Luminous extinction coefficient
B Parameter defining zenith luminance (see Kittler and others [2012], table 5.2)
C Exponential parameter defining zenith luminance (see Kittler and others [2012], table 5.2)
D Exponential parameter defining zenith luminance (see Kittler and others [2012], table 5.2)
E Parameter defining zenith luminance (see Kittler and others [2012], table 5.2)
E_{eo} Solar constant (maximal extraterrestrial irradiance equal to 1366.1 W/m^2);
$E_{eo,h}$ Extraterrestrial horizontal irradiance
E_{ed} Sky/diffuse horizontal irradiance at ground level
E_{eg} Global horizontal irradiance at ground level
E_{es} Solar horizontal irradiance at ground level
E_{vo} Luminous solar constant (maximal extraterrestrial illuminance equal to 133.8 klx)
$E_{vo,h}$ Extraterrestrial horizontal illuminance
E_{vd} Skylight/diffuse horizontal illuminance at ground level
E_{vg} Global horizontal illuminance at ground level
E_{vs} Sunlight/direct horizontal illuminance at ground level
γ_s Solar altitude
L_{vZ} Zenith luminance
m Relative optical air mass
T_L Atmospheric turbidity factor in the whole solar energy spectrum after Linke [1922]
T_v Luminous turbidity factor
X Auxiliary parameter defining zenith luminance equal to $(\sin\gamma_s)^C/(\cos\gamma_s)^D$
ZL Zenith luminance factor after IESNA *Lighting Handbook* [IESNA 2000]

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