22 STANDARDISATION OF OUTDOOR CONDITIONS FOR THE CALCULATION OF DAYLIGHT FACTOR WITH CLEAR SKIES

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Daylight conditions under clear sky comprise direct sunlight and also the diffuse skylight caused by the diffusion of sunlight in the atmosphere.
Similarly to overcast sky conditions, the cloudless sky, functioning as a large area source, is of great importance for the daylighting of interiors, as its light reaches even to such places in the interior, to which direct sunlight cannot penetrate at a given time of the day owing to the orientation and system of apertures. Theoretically, let us imagine that the sun is obstructed by a small cloud, which moves slowly with the apparent motion of the sun.
An illuminating engineer's characteristics of the clear sky for the purposes of daylight calculation methods must be based on the present knowledge of atmospheric optics, and such typical conditions of the clear sky must be chosen as to allow its simple definition as a standard area source. First of all, therefore, it is necessary to study the clear sky luminance distribution and to determine its typical luminance pattern, which changes according to the motion of the sun.
The sunlight diffusion process in the atmosphere and its final effect, which causes a certain luminance distribution of the clear sky, may be characterized by the diffusion indicatrix. The hitherto relatively scarce knowledge of this phenomenon in detail is due to the difficulties of defining the complex multiple diffusion and reflection of sunlight on the gas and aerosol particles in the actual turbid environment of the atmosphere.
The diffusion of light caused by gas molecules in a so-called ideally clear atmosphere has been characterized by Rayleigh's relative diffusion indicatrix (1):

$$f(\gamma) = 1 + \cos^2 \gamma \qquad (1)$$

where γ is the arbitrary diffusion angle or the angular distance of the considered sky element from the sun.
It has been proved that this ideal diffusion indicatrix can be real in the extraordinary conditions of very dry, absolutely clear and transparent air. For instance, a mountaineering expedition of the Soviet Academy of Sciences measured an indicatrix very similar to that of Rayleigh at an altitude of 3,200 and 4,000 metres (10,500 and 13,100 ft.) on the Elbrus in the Caucasus, the relative humidity of the air being 26%, the air temperature 5° C (41° F), the visibility distance about 220 km (137 miles) (2). A similar indicatrix was measured by Fesenkova (3) in a desert of the South Pribalkhash at an altitude less than 400 metres (1,310 ft.) which was closer to Rayleigh's indicatrix than the one measured on the Kumbel mountain (3,140 m or 10,300 ft high) in the region of Alma Ata. This fact proves the great significance of the absence of aerosols and chiefly of small drops of water and humidity in the atmosphere.
The investigations of diffusion indicatrices in the lowest layer of the atmosphere, based on 715 measured cases (2) have shown that 10 general types can be distinguished. For our purposes only the first seven types for a visibility distance over 5 km (3.5 miles) are of interest (see figure 22.1, where zero indicates the ideal indicatrix of Rayleigh).
Apart from the first type (according to measurements made on the Elbrus), all indicatrices are extended in the direction of the sunbeam. The greater is the

Fig. 22.1

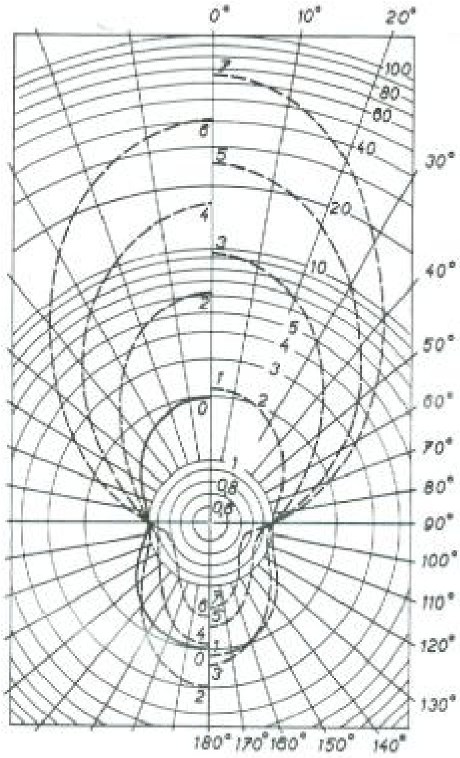
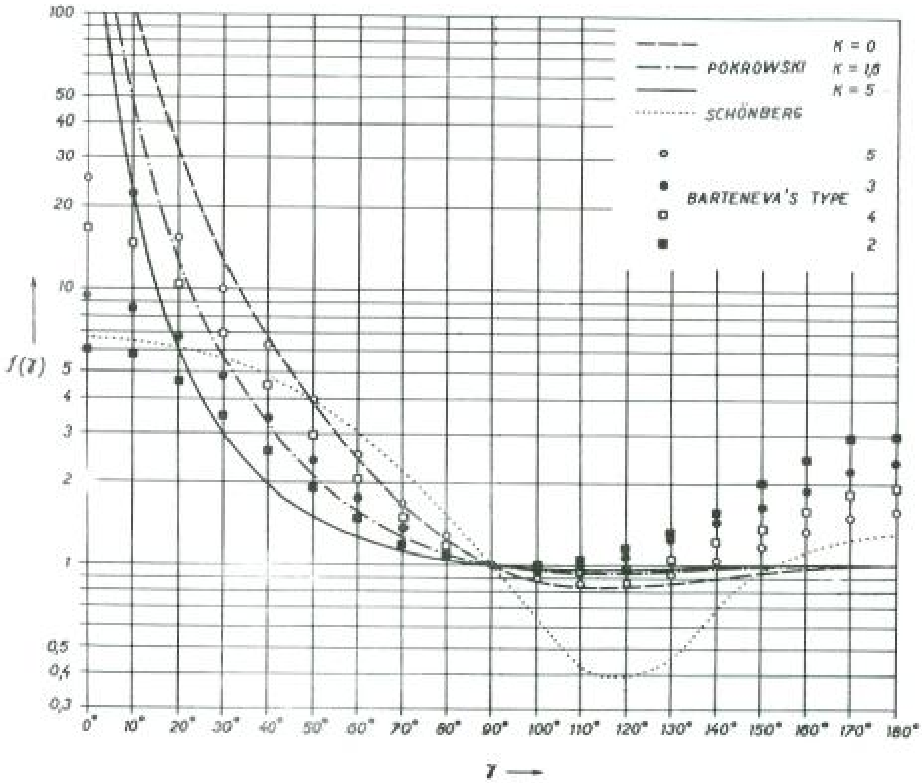


Fig. 22.2



extension in the forward direction, the smaller is the part at the rear. This extension of the indicatrix in a forward direction is typical for turbid and aerosol containing environment (Mie's effect (4)).
Schönberg (5) recommended that the atmospheric indicatrix be defined by a function:

$$f(\gamma) = 1 + p \cos \gamma + q \cos^2 \gamma \qquad (2)$$

According to his investigation under foggy conditions he chose the parameters p = 2.7 and q = 3.0. In the case when p = 0 and q = 1 he obtained the indicatrix of Raleigh. Later on several authors pointed out, that an indicatrix according to equation (2) does not hold for a clear sky.
At the same time, in the literature of the twenties we can find definitions of the luminance pattern of a clear sky, which assume a certain standard diffusion indicatrix. For example, the formula of Pokrowski (6) considers a relative indicatrix for the secondary scattering constant K = 0:

$$f(\gamma) = \frac{1 + \cos^2 \gamma}{1 + \cos \gamma} \qquad (3)$$

or for K = 5:

$$f(\gamma) = \frac{1}{6} \left(\frac{1 + \cos^2 \gamma}{1 - \cos \gamma} + 5 \right) \qquad (4)$$

Analogically the indicatrix for K = 1,6 is defined by the author (7). A comparison of these mathematically defined indicatrices with those of the 2nd, 3rd, 4th and 5th type by Barteneva can be seen on figure 22.2. It is evident, that neither Schönberg's nor Pokrowski's indicatrices correspond fully with the measured averages. Pokrowski's indicatrices give for $\gamma = 0$ an unreal, infinite value, while near the angle 180° the curves are too low.
Krat (8) studied the correction of Schönberg's functional relation for clear sky conditions. On the basis of his own measurements he recommended the formula:

$$f(\gamma) = 1 + p \left[\exp(-3\gamma) - 0{,}009 \right] + q \cos^2 \gamma \qquad (5)$$

By introducing a new function to the parameter p, the increasing luminance of the sun aureola can be adequately expressed. With the increase of the p value the indicatrix is extended forward.
On the basis of more than 180 data measured by Krat, it can be stated that parameter p changes under clear sky conditions within the limits 1.9 to 17.6 with an average of 10.46 and value q varies from -0,59 to 1.46 with an average 0.48. The three curves on figure 22.3 are characteristic:

1) dashed line for p = 15, q = 0.33;

2) full line - average for p = 10, q = 0.45;

3) dash-dotted line for p = 5, q = 0.8.

Similarly to figure 22.1 these indicatrices form a transition to Rayleigh's ideal indicatrix (dotted line on figure 22.3), which is given by formula (5) when p = 0 and q = 1.
By a suitable choice of these two parameters it is possible to follow the chosen indicatrix type or the results of certain particular measurements. For example, figure 22.4 shows the curve according to formula (5) when p = 5, q = 0.55 that corresponds to Fesenkova's measurements for the wave length 546 mu (Ref. 3 p. 146). Apart from this, figure 22.4 gives the data of Boldyrev (9) derived from Dorno's measurements (10) which can be expressed by formula (5), when p = 10 and q = 0.35. This last curve is in approximate agreement with the mean data measured by Krat.
In the author's opinion, the atmospheric conditions for a C.I.E. standard clear sky could be satisfactory defined by the diffusion indicatrices within a tolerance zone between the curves:

$$f(\gamma) = 0{,}955 + 5 \exp(-3\gamma) + 0{,}8 \cos^2 \gamma \qquad (6)$$

and

$$f(\gamma) = 0{,}865 + 15 \exp(-3\gamma) + 0{,}33 \cos^2 \gamma \qquad (7)$$

while the standard indicatrix:

$$f(\gamma) = 0{,}91 + 10 \exp(-3\gamma) + 0{,}45 \cos^2 \gamma \qquad (8)$$

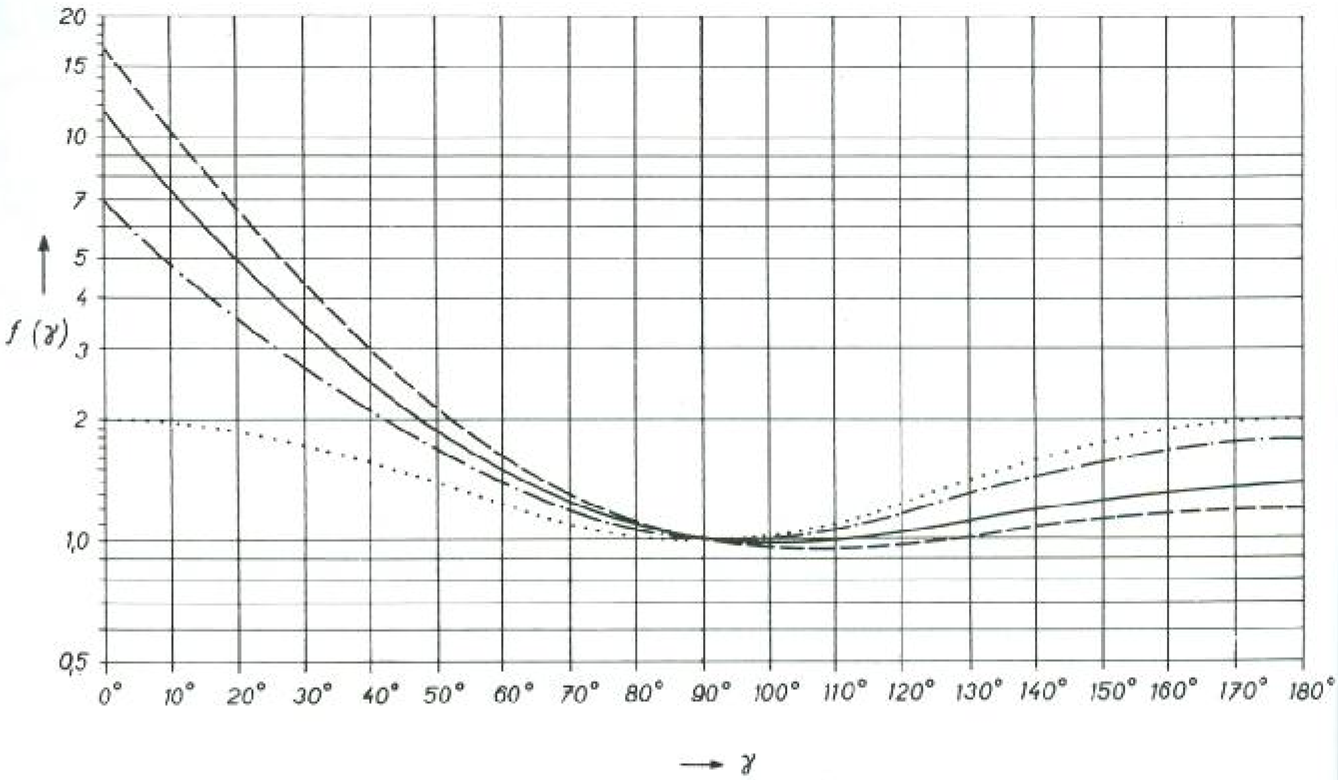


Fig. 22.3

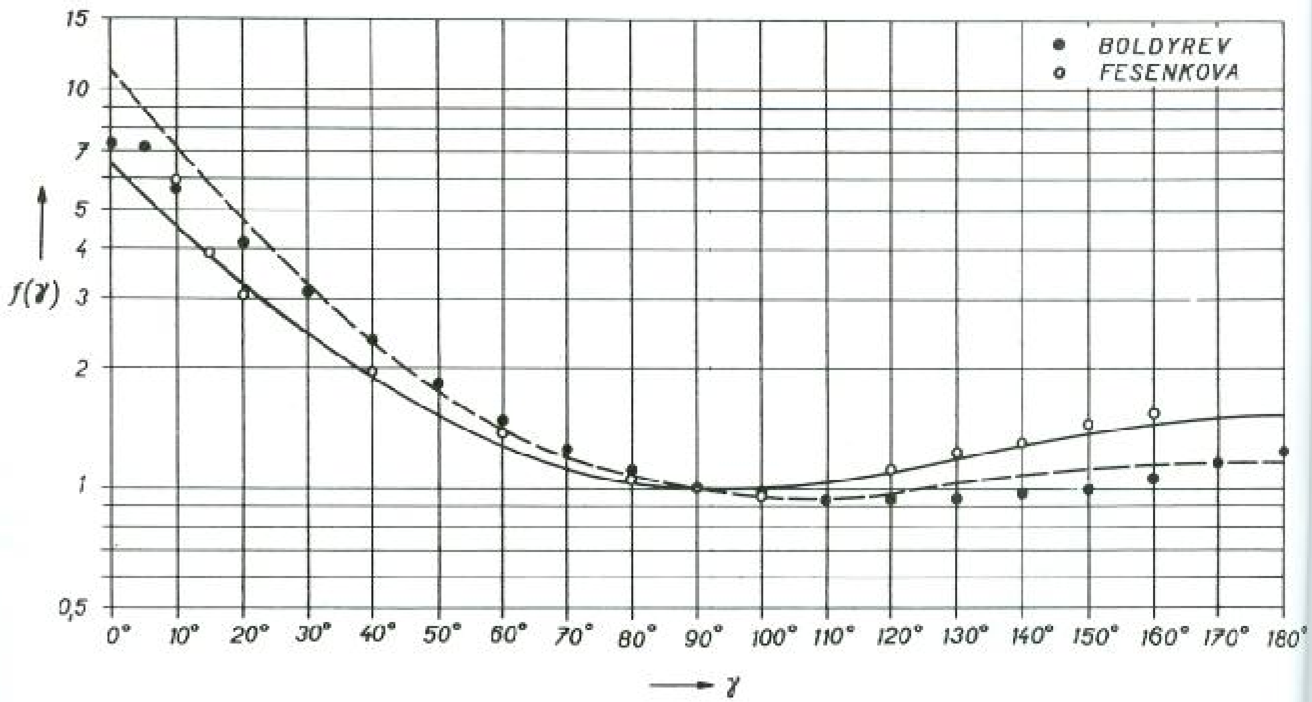


Fig. 22.4

would correspond to the ideal luminance distribution, which would serve as a basis for the calculation of Daylight Factor under clear sky.
In table 22.1 these curves are given by numeric values, while their graphical representation is already shown on figure 22.3.
These standard indicatrices determine, in addition to the characteristics of the diffusion process, also its correlation with certain assumed atmospheric transparencies or air pollutions and their deviations, which could occur in residential areas under the standard conditions.

Table 22.1 Values of f (γ) determining standard atmospheric diffusion indicatrices.

γ in degrees	f (γ) according to formula: (6)	(8)	(7)
0	6.755	11.360	16.195
5	5.599	9.053	12.736
10	4.693	7.270	10.071
20	3.416	4.816	6.420
30	2.594	3.326	4.230
40	2.040	2.406	2.906
50	1.650	1.825	2.096
60	1.371	1.455	1.596
70	1.177	1.219	1.288
80	1.055	1.075	1.102
90	1.000	1.000	1.000
100	1.006	0.977	0.955
110	1.064	0.994	0.951
120	1.164	1.041	0.976
130	1.291	1.107	1.018
140	1.428	1.181	1.069
150	1.557	1.251	1.118
160	1.663	1.310	1.160
170	1.732	1.348	1.187
180	1.755	1.360	1.196

Under these assumptions the relative luminance of an arbitrarily located sky element P can, for practical engineering purposes, be expressed with sufficient accuracy by two functional relations:

$$\frac{L_P}{L_Z} = \frac{\varphi(\zeta_P)}{\varphi(0^0)} \cdot \frac{f(\gamma)}{f(z_\odot)} \qquad (9)$$

where L_Z is the zenith luminance of the clear sky,

$z_\odot$ the angular distance of the sun from the zenith,

ζ_P the angular zenith distance of the sky element under consideration,

γ is the angular distance of the sky element from the sun defined by general equation:

$$\cos\gamma = \cos z_\odot \cdot \cos\zeta_P + \sin z_\odot \cdot \sin\zeta_P \cdot \cos A_P \qquad (10)$$

where A_P is the azimuthal angle of the meridian of the sky element P from the momentary sunmeridian.

In the case that we have to determine the luminance distribution on the sun meridian only, we do not have to apply formula (10) as the angle γ can be easily found. When calculating the luminance pattern of the whole sky hemisphere, for solar altitudes at 10^0 intervals, the γ value can be read from table 22.2. The function f(γ) and its determination have already been discussed. The second functional relation φ in formula (9), which depends on ζ_p, has been studied by Pokrowski and expressed by him in a simple form:

$$\varphi (\zeta_p) = K_o \left[1 - \exp (-P \sec \zeta_p)\right] \qquad (11)$$

where P is a primary scattering constant, determined experimentally by the author with a value 0,32.

Table 22.2 Angular distance of the sky element (γ) from the sun in degrees

A_p	Angle ζ_p									
	0	10	20	30	40	50	60	70	80	90
	$z_{\odot} = 0$									
for all values	0	10	20	30	40	50	60	70	80	90
	$z_{\odot} = 10$									
0	10	0	10	20	30	40	50	60	70	80
15	10	2	10	20	30	40	50	60	70	80
30	10	5	12	21	31	41	51	61	71	81
45	10	7	14	23	33	43	53	63	73	82
60	10	10	17	26	35	45	55	65	75	85
75	10	12	19	28	38	48	57	67	77	87
90	10	14	22	31	41	50	60	70	80	90
105	10	15	24	33	43	53	63	72	82	93
120	10	17	26	35	45	55	65	75	85	95
135	10	18	27	37	47	57	67	77	87	98
150	10	19	29	38	48	58	68	78	88	99
165	10	19	29	39	49	59	69	79	90	100
180	10	20	30	40	50	60	70	80	90	100
	$z_{\odot} = 20$									
0	20	10	0	10	20	30	40	50	60	70
15	20	10	5	11	21	31	40	50	60	70
30	20	12	9	15	24	33	43	53	62	72
45	20	14	15	20	28	37	47	56	66	76
60	20	17	19	25	33	42	51	61	70	80
75	20	19	24	30	39	47	56	66	75	84
90	20	22	27	35	43	52	61	71	80	90
105	20	24	31	39	48	57	66	76	85	96
120	20	26	34	43	52	61	71	80	90	100
135	20	27	36	46	55	65	74	84	95	104
150	20	29	38	48	58	67	77	87	98	108
165	20	29	39	49	59	69	79	89	100	110
180	20	30	40	50	60	70	80	90	100	110
	$z_{\odot} = 30$									
0	30	20	10	0	10	20	30	40	50	60
15	30	20	11	7	13	22	31	41	51	61
30	30	21	15	14	19	27	36	45	54	64
45	30	23	20	22	27	34	42	51	60	69

A_P	Angle ζ_P 0	10	20	30	40	50	60	70	80	90
	$z_\odot$ = 30 contd.									
60	30	26	25	28	34	41	49	57	66	75
75	30	28	30	35	41	49	56	65	73	82
90	30	31	35	41	48	56	64	72	81	90
105	30	33	39	46	54	62	71	79	88	98
120	30	35	43	51	59	68	77	86	96	105
135	30	37	46	55	64	73	82	93	102	111
150	30	38	48	57	67	76	86	97	107	116
165	30	39	49	59	69	79	89	99	109	119
180	30	40	50	60	70	80	90	100	110	120
	$z_\odot$ = 40									
0	40	30	20	10	0	10	20	30	40	50
15	40	30	21	13	9	14	22	32	41	51
30	40	31	24	19	19	23	30	38	47	56
45	40	33	28	27	28	32	39	46	54	62
60	40	35	33	34	37	42	48	55	63	71
75	40	38	39	41	46	51	58	65	72	80
90	40	41	43	48	54	60	67	74	82	90
105	40	43	48	54	61	67	76	83	92	100
120	40	45	52	59	67	75	83	93	101	109
135	40	47	55	64	72	81	91	100	109	118
150	40	48	58	67	76	86	96	106	115	124
165	40	49	59	69	79	89	99	109	119	129
180	40	50	60	70	80	90	100	110	120	130
	$z_\odot$ = 50									
0	50	40	30	20	10	0	10	20	30	40
15	50	40	31	22	14	11	15	23	32	42
30	50	41	33	27	23	22	26	32	40	48
45	50	43	37	34	32	34	37	43	49	57
60	50	45	42	41	42	45	49	54	60	67
75	50	48	47	49	51	55	60	66	72	78
90	50	50	52	56	60	65	71	77	83	90
105	50	53	57	62	68	74	81	88	95	102
120	50	55	61	68	75	83	91	99	106	113
135	50	57	65	73	81	91	99	107	115	123
150	50	58	67	77	86	96	105	114	127	132
165	50	59	69	79	89	99	109	119	129	138
180	50	60	70	80	90	100	110	120	130	140
	$z_\odot$ = 60									
0	60	50	40	30	20	10	0	10	20	30
15	60	50	40	31	22	15	13	16	24	33
30	60	51	43	36	30	26	25	28	34	41

A_P	Angle ζ_P									
	0	10	20	30	40	50	60	70	80	90
	z_Θ = 60 contd.									
45	60	53	47	42	39	37	38	41	46	52
60	60	55	51	49	48	49	51	54	59	64
75	60	57	56	56	58	60	63	67	72	77
90	60	60	61	64	67	71	75	80	85	90
105	60	63	66	71	76	81	86	93	98	103
120	60	65	71	77	83	91	98	104	110	116
135	60	67	74	82	91	99	107	114	122	128
150	60	68	77	86	96	105	114	123	131	139
165	60	69	79	89	99	109	119	128	138	147
180	60	70	80	90	100	110	120	130	140	150
	z_Θ = 70									
0	70	60	50	40	30	20	10	0	10	20
15	70	60	51	41	32	24	17	14	18	25
30	70	61	53	45	38	34	29	28	31	36
45	70	63	57	51	46	43	42	42	44	48
60	70	65	61	58	56	55	55	56	59	62
75	70	67	66	65	65	66	68	70	73	76
90	70	70	71	73	75	78	80	83	87	90
105	70	73	76	80	84	88	92	96	100	104
120	70	75	81	86	92	98	104	109	114	118
135	70	77	85	92	100	107	114	121	127	132
150	70	79	87	96	105	114	122	130	138	144
165	70	80	89	99	109	119	128	137	147	155
180	70	80	90	100	110	120	130	140	150	160
	z_Θ = 80									
0	80	70	60	50	40	30	20	10	0	10
15	80	70	60	51	41	32	24	17	14	17
30	80	71	62	54	47	40	34	30	29	31
45	80	73	66	60	54	49	46	44	44	45
60	80	75	70	66	63	60	59	58	59	60
75	80	77	75	73	72	72	72	72	73	75
90	80	80	80	81	82	83	85	86	88	90
105	80	82	85	92	92	95	98	101	103	105
120	80	85	91	96	101	106	110	114	118	120
135	80	87	95	102	109	115	122	127	131	135
150	80	88	98	107	115	123	131	138	145	149
165	80	89	100	109	119	129	138	147	156	163
180	80	90	100	110	120	130	140	150	160	170
	z_Θ = 90									
0	90	80	70	60	50	40	30	20	10	0
15	90	80	70	61	51	42	33	24	17	15

A_P	Angle ζ_P									
	0	10	20	30	40	50	60	70	80	90
	$z_\odot = 90$ contd.									
30	90	81	72	64	56	48	41	35	31	30
45	90	82	76	69	62	57	52	48	45	45
60	90	85	80	75	71	67	64	61	60	60
75	90	87	84	82	80	78	77	75	75	75
90	90	90	90	90	90	90	90	90	90	90
105	90	93	96	98	100	102	103	105	105	105
120	90	95	100	105	109	113	116	119	120	120
135	90	98	104	111	118	123	128	132	135	135
150	90	99	108	116	124	132	139	145	149	150
165	90	100	110	119	129	138	147	156	163	165
180	90	100	110	120	130	140	150	160	170	180

Pokrowski did not give the value of the parameter K_o, but, however, this can be eliminated by using the ratio of two φ functions in formula (9), thus

$$\frac{\varphi(\zeta_P)}{\varphi(0^\circ)} = \frac{1 - \exp(-P \sec \zeta_P)}{1 - \exp(-P)} \qquad (12)$$

When comparing the results of formula (12) if P = 0.32 with Boldyrev's data (see figure 22.5) a relatively good agreement can be found, though close to the horizon the curve for P = 0.265 is even better.
In conclusion, for the definition of a standard luminance distribution of the C.I.E. clear sky, the following formula is recommended:

$$\frac{L_P}{L_Z} = \frac{\left[1 - \exp(-0.32 \sec \zeta_P)\right]\left[0.91 + 10 \exp(-3\gamma) + 0.45 \cos^2 \gamma\right]}{0,27385\left[0.91 + 10 \exp(-3 z_\odot) + 0.45 \cos^2 z_\odot\right]} \qquad (13)$$

At first sight this formula may seem to be too complicated, however, for the principal solar altitudes, when it is possible to use table 22.2, the f and φ functions can be read from figure 22.3, or table 22.1 and figure 22.5 respectively.
The use of the formula can be illustrated by the following example:
It is necessary to find the relative luminance on the sun meridian in two places, defined by $\zeta_P = 40^\circ$, when the altitude of the sun is 60° i.e. $z_\odot = 30^\circ$.
According to figure 22.5 will be $\frac{\varphi(\zeta_P)}{\varphi(0^\circ)} = 1.25$ according to figure 22.3 or table 22.1 : $f(\gamma = 10^\circ) = 7.27$, $f(\gamma = 70^\circ) = 1.22$ and in both cases $f(z_\odot = 30^\circ) = 3.33$. On the sun meridian, at a point 10° from the sun the relative luminance will be

$$\frac{L_P}{L_Z} = 1.25 \ \frac{7.27}{3.33} = 2.72;$$

whereas at a point 70° from the sun it will be

$$\frac{L_P}{L_Z} = 1.25 \ \frac{1.22}{3.33} = 0.45$$

In a similar way we have found the relative sky luminance distribution on the sun meridian in cases:
a) for sun altitude 0°, i.e. $z_\odot = 90^\circ$ on figure 22.6
b) for sun altitude 30°, i.e. $z_\odot = 60^\circ$ on figure 22.7
c) for sun altitude 60°, i.e. $z_\odot = 30^\circ$ on figure 22.8
d) for the sun in the zenith, $z_\odot = 0^\circ$ on figure 22.9.
The tolerance zone corresponding to the diffusion indicatrices from equations

Fig. 22.7

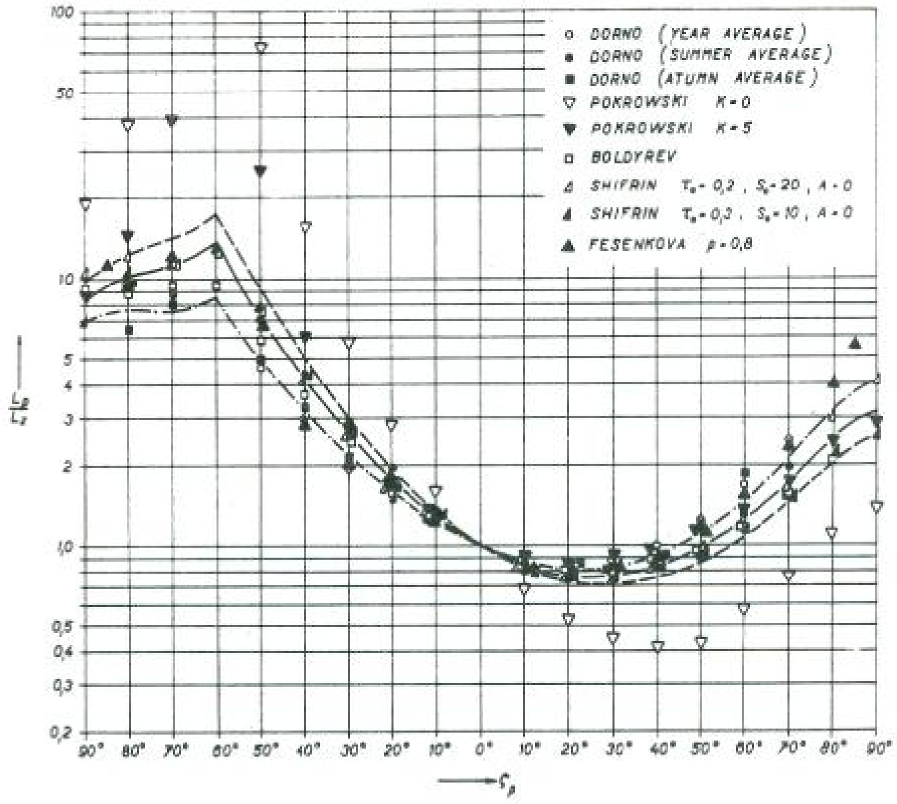


Fig. 22.8

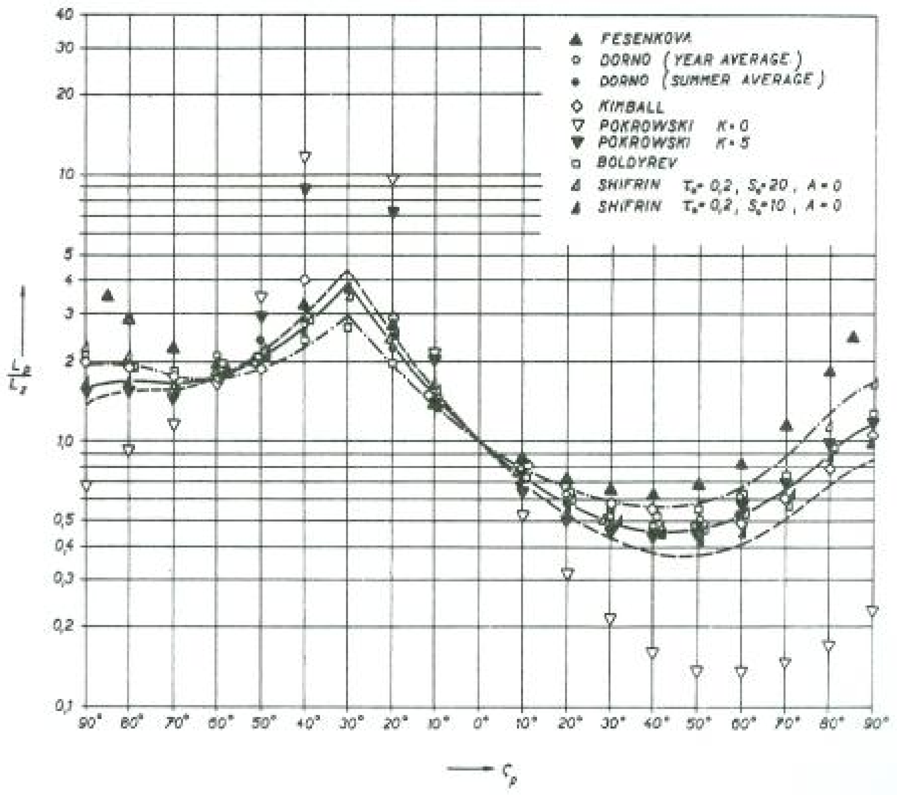


Fig. 22.9

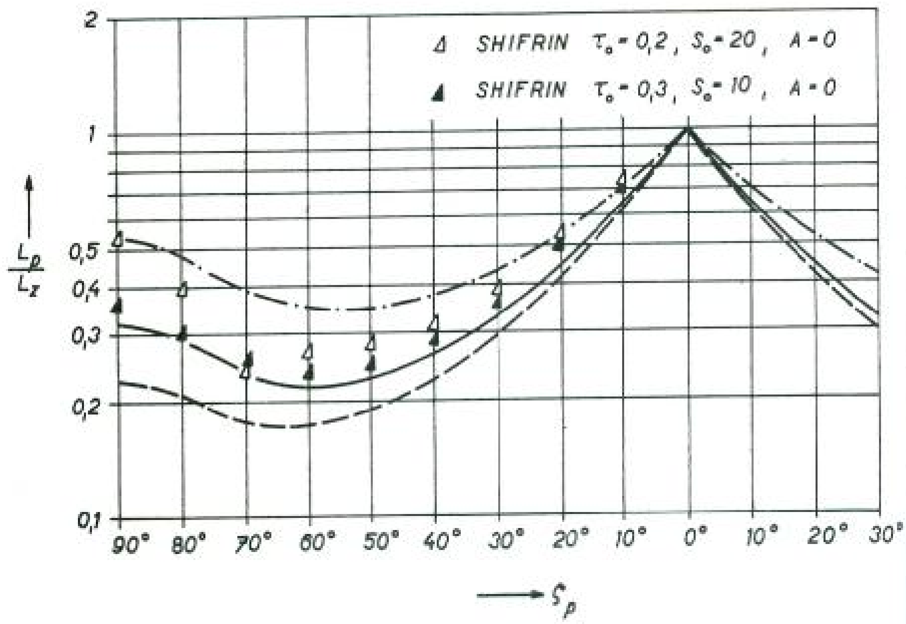
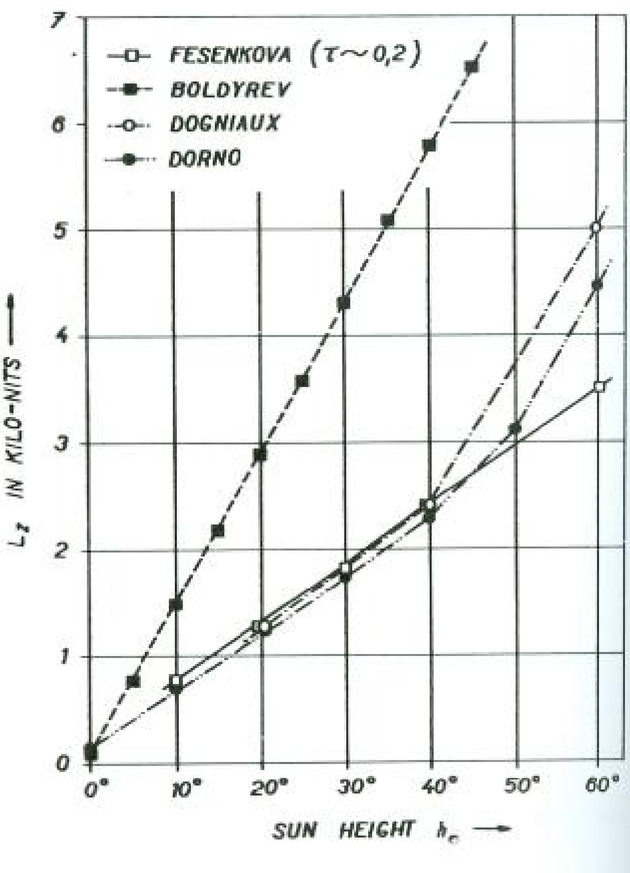


Fig. 22.10



(6) and (7) can be seen on these figures as well as the ideal luminance distribution from equation (13) (indicated by a full line). For the sake of comparison the data of other authors (3), (6), (9 to 13) are shown where available as well.

Having decided upon the standardizing formula, it will be possible to prepare tables or graphs which would give the computed relative luminance values for the main sun heights.

To be able to determine the luminance distribution in absolute values on the basis of the relative distribution, it will be necessary to define the dependency of the zenith luminance on the solar altitude in such transparent atmosphere, which would correspond to a standard indicatrix. With regard to this fact a further investigation and the summarization of the measured data in various geographic regions will be necessary. According to preliminary data taken from available literature (see figure 22.10) this dependency seems to have the form of a simple relation.

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