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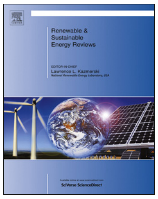

Scattered sunlight determining sky luminance patterns

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ABSTRACT

During the first daylight photometry studies in the 18th Century three fundamental source types of daylight were characterised as direct sunlight, clear sky luminance distribution and overcast sky luminance uniformity. For minimum illumination and simplicity the last became the basic state to determine computational methods and practical graphical tools to evaluate daylight distribution especially in poorly naturally lit interiors. The simultaneously present sunlight and skylight under cloudless or cloudy conditions attracted the research of sky luminance distributions caused by the scattering of sunbeams within the atmosphere. The arbitrary real sky luminance patterns influenced by air molecule and aerosol turbidity as well as cloudiness were studied and measured to determine the relatively complex their changes. The historical search after the interrelated definition and mathematical expression of gradation and scattering indicatrix functions is explained in the gradually evolving progress in this paper.

Recently in sustainable solar energy utilisation and daylighting studies via building apertures and photovoltaic panels take into account also radiance and luminance gains from the sky vault under any weather conditions. The luminance distribution on homogeneous sky types is today based on mathematically defined ISO/CIE standards of relative gradation and indicatrix functions in the range between overcast and cloudless skies. The review of the development and the different methodologies of categorising typical sky luminance patterns is discussed and summarised in this paper.

Contents

1. Introduction

In this paper a special review of two indicatrix and gradation concepts is documented and compared in the development of sky radiance and luminance distribution on the sky vault which seems to be crucial within the above research chain fundamentally influenced by the scattering trend of originally parallel sun beams and the gradation trends corresponding to the optical air mass of the atmosphere.

As soon as Bouguer [1] and Lambert [2] published their seminal books on photometry it was evident that exterior daylight

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illuminance on the unobstructed horizontal plane depends on direct sunlight, clear sky luminance distribution or overcast uniform luminance covering the sky vault. Consequently the interior illuminance was caused by the influence of penetrated sky luminance "seen" from any interior plane element within the window solid angle related to its projection onto the illuminated plane element. In a mathematical expression the skylight illuminance in the interior $E_{v,i}$ is given by the integral/sum of luminances $L_{v,\chi zi}$ of every sky element within the aperture solid angle ω projected onto the illuminated plane according with its incidence angle i, i.e. after cos i, thus

$$E_{v,i} = \int_{\omega=1}^{\omega=i} L_{v,\chi z} \cos\ i\ d\omega_i \tag{1}$$

In spite of some luminance measurements of the clear sky distribution by Bouguer his handwritten notes and measured values could not be reproduced in his posthumous book. Therefore the Lambertian uniform and unity luminance sky model was taken as representing minimal and critical illuminance levels for window design and for the basis criteria and standards in daylighting applications.

2. Uniform and gradual luminance distributions on overcast skies

Studies and definitions of the solid angle and its projections were relatively easily solved by applying principles of spherical geometry or trigonometry function by Lambert and some of his followers, and they avoided the complex problems of various sky luminance patterns by a simple assumption that the sky luminance on the whole sky vault was uniform and equal to unity. In fact, such a sky seemed to be observed under densely overcast skies and/or with fog in autumn and winter months. However, under cloudless and clear skies this assumption was quite far from reality as Bouguer had already measured some years ago. Nobody dared to repeat such a tedious measurement by a subjective luminance metre, so the Lambert sky was used until the first years of the 20th Century. The first steps to test the Lambert sky using a newer photometer were taken by Schramm [3] and Kähler [4] who later introduced a sine gradation 1:3 empirical formula based on analysis of his measurements:

$$\frac{L_{v\varepsilon}}{L_{vZ}} = 1 + 2\ \sin\ \varepsilon \tag{2}$$

where $\frac{L_{v\varepsilon}}{L_{vZ}}$ is the ratio of luminance in any sky element in elevation ε to the luminance in zenith, ε the angular altitude of the sky element in deg.

After more than 30 years Moon and Spencer [5] rediscovered and tested Eq. (2) and proposed its international adoption in the form of cosines of the element zenith angle Z to suit the normalisation to unit zenith luminance that was accepted in the CIE [6] recommendation:

$$\frac{L_{vz}}{L_{vZ}} = \frac{1}{3}(1 + 2\ \cos\ Z) \tag{3}$$

where $\frac{L_{vz}}{L_{vZ}}$ is the ratio of luminance in any sky element in angular zenith distance Z to the luminance in the zenith, i.e. in the range of 1/3: 1.

Thus the sky gradation from horizon to zenith 1/3:1 was standardised and all calculations and skylight graphs or nomograms in practical use had to be corrected appropriately [7].

Luminance distribution under an overcast sky with snow covered ground was later proposed by Petherbridge [8] with the gradation 1/2: 1 in Fig. 1:

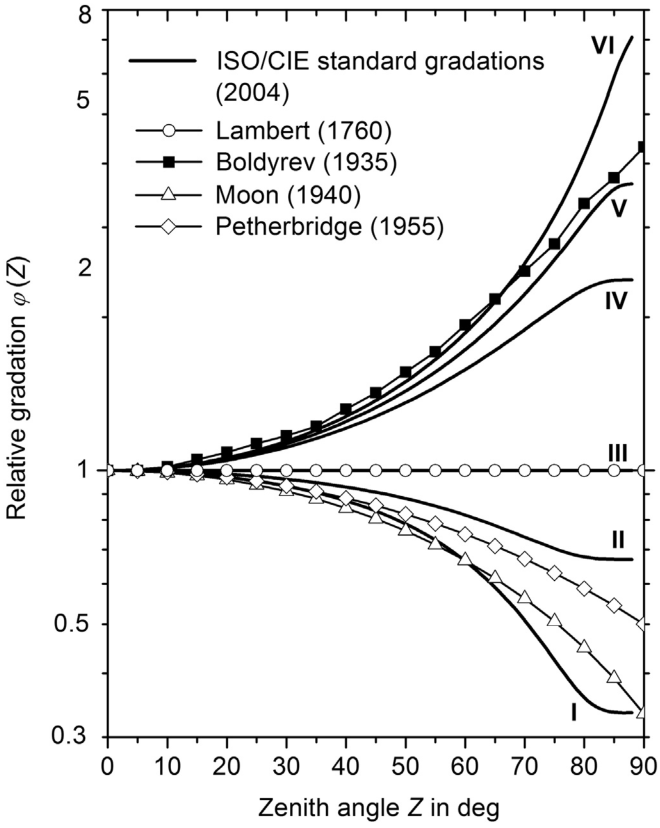

Fig. 1. Comparison of different gradation functions.

$$\frac{L_{vz}}{L_{vZ}} = \frac{1}{2}(1 + \ \cos\ Z) \tag{4}$$

3. Seminal studies of sunlight scattering in cloudless atmospheres

However, in absence of reference sky luminance patterns under clear sky conditions was felt as a serious problem. In Bouguer's posthumous book [1] attention was drawn to his luminance measurements on a clear sky, but unfortunately there were no corroborating values detected from his original notes. Due to imprecise data from subjective luminance instruments with relatively wide acceptance angles very few investigators tried to measure the scattered sunlight and to determine the relative or absolute indicatrix covering the spatial distribution influencing the sky pattern.

The relative scattering indicatrix $f(\chi)$ is defined by the luminance in any direction to that which is perpendicular to the original sunbeam direction

$$f(\chi) = \frac{L_\chi}{L_{90^\circ}} \tag{5}$$

In some studies instead of the relative function the absolute indicatrix value $s(\chi)$ which is significantly lower is used as

$$s(\chi) = \frac{f(\chi)}{K} = \frac{f(\chi)}{2\pi \int_0^\pi \sin\ \chi\ d\chi} \tag{6}$$

After the integration K values for the standard in dicatrices are:

for ISO/CIE indicatrix 1: $K = 4\pi = 12.57$;
for ISO/CIE indicatrix 2: $K = 14.71$;

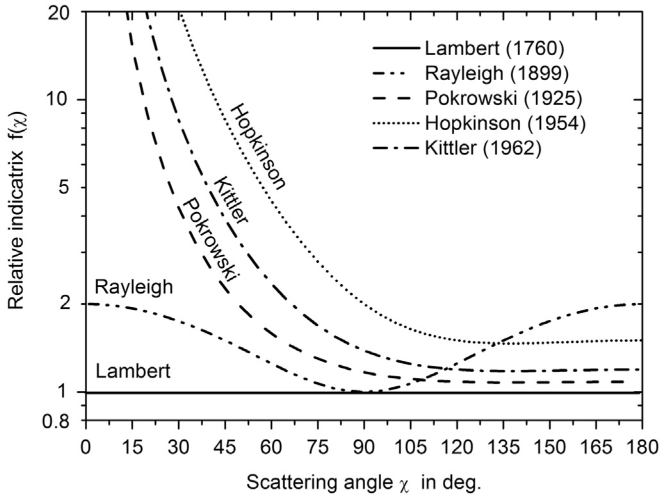

Fig. 2. Semi-logarithmic plot of elongated indicatrix courses after Eq. (8) with comparison to Rayleigh's after Eq. (7) and Lambertian with unity indicatrix.

for ISO/CIE indicatrix 3: $K = 16.92$;
for ISO/CIE indicatrix 4: $K = 19.61$;
for ISO/CIE indicatrix 5: $K = 22.07$;
for ISO/CIE indicatrix 6: $K = 26.55$.

Note that the absolute indicatrices were applied e.g. by Nagata [9].

The gaseous and water content of the atmosphere as well as the cloud formation and intervention complicated the complexity of real sky luminance patterns. The simplified cloudless case including the sky transformation to a blue colour was explained by Lord Rayleigh (John William Strutt), [10] defined the relative scattering indicatrix function $f(\chi)$ mathematically for the absolutely clean atmosphere containing only air molecules by

$$f(\chi) = 1 + \cos^2\chi \tag{7}$$

where χ is the arbitrary angle from the original sun beam direction. The indicatrix course can be represented in a semi-logarithmic diagram, where the original beam relative luminance direction is at $\chi=0°$, normalising luminance at $\chi=90°$ while the backward/tail luminance is at 180° (Fig. 2).

However, in the cloudless atmosphere even a pollution by aerosols of different size and structure distort the indicatrix profile while the Rayleigh indicatrix is equally elongated on the forward and backward directions. Mie [11] found that parallel beams in a turbid solution penetrated and elongated elongated the indicatrix profile, further increased turbidity and reduced media transmittance and caused a uniform, close to unity diffusion effect, i.e. the absolute diffusion assumed by Lambert under densely overcast and foggy skies. Pokrowski [12] studied the indicatrix under real cloudless skies and characterised its elongation adding altering parameters in his relative function

$$f(\chi) = \frac{1}{1+A}\left(\frac{1+\cos^2\chi}{1-\cos\chi} + A\right) \tag{8}$$

and he found the best fit when $A=5$. Later Hopkinson [13] in accordance with his Stockholm subjective luminance measurements proposed the $A=0$, while Kittler [14] after a Bratislava measurement suggested $A=1.6$.

While the indicatrix is very elongated and prevailing in the forward direction, the tail course is almost horizontal except after Rayleigh when even under cloudless skies the air molecules are present. However, such extreme indicatrix elongations were possibly caused by the rather wide acceptance angles of luminance metres allowing direct solar disc luminance entering the luminance tube. To avoid such an error a team of Russian astrophysicists at the Moscow University observatory measured the luminance distribution on a cloudless sky further from the sun disc rim using the subjective Lummer luminance metre [15]. Assuming a single scatter their new results discovered two separate individual functions of gradation and of scattering indicatrix that influenced mainly the clear sky luminance pattern. Thus a first practical proposal was made by Boldyrev [16] recommending the normalised luminance ratio be expressed according to sky element γ and solar altitude γ_s or zenith angles Z and Z_s in the relative formula of their functions:

$$\frac{L_{v\chi}}{L_{vZ}} = \frac{f(\chi)\varphi(\gamma)}{f(90^\circ-\gamma_s)\varphi(90^\circ)} = \frac{f(\chi)\varphi(Z)}{f(Z_s)\varphi(0^\circ)} \tag{9}$$

where $\frac{L_{v\chi}}{L_{vZ}}$ is the sky luminance in any arbitrary sky element with the spherical angular distance χ from the sun position normalised to zenith luminance L_{vZ}; $f(\chi)$ and $f(90°\text{-}\gamma_s)$ or $f(Z_s)$ are scattering indicatrix functions dependent either on angle χ or Z_s respectively; $\varphi(\gamma)$ and $\varphi(90°)$ or $\varphi(Z = 0°)$ are luminance gradation functions respectively.

Both Fesenkov and Boldyrev were not able to determine these functions in mathematical form but used only tabulated values after few measurement results. Note also, that while Fesenkov's values were given in a relative indicatrix course, Boldyrev's values had to be normalised to unity zenith as presented in Fig. 3. Inserted are also Peraudeau's model of five sky indicatrixes which will be discussed in Section 4.2.

However, after astronomic studies by Henyey and Greenstein [17] their phase function is often used for a more turbid clear sky with aerosol content as relative indicatrix function

$$f(\chi) = \left[\frac{1-g^2}{(1+g^2-2g\cos\chi)^{1.5}}\right] : \left[\frac{1-g^2}{(1+g^2)^{1.5}}\right] \tag{10}$$

Although the original paper used Eq. (10) in the astrophysical form of a phase function (see [8], chapter 4) it is sometimes applied now also as a relative indicatrix in the range from isotropic distribution for overcast skies when $g=0$, and various elongated indicatrices with $g=0.3–0.67$. (Fig. 4). Recently for the clear and aerosol turbid sky types a combination of Rayleigh's and Henyey–Greenstein indicatrices in the proportions 1:1 and 1:2 respectively was recently suggested by Budak and Smirnov [18]. However, that proposal was founded on simulation to represent CIE sky types

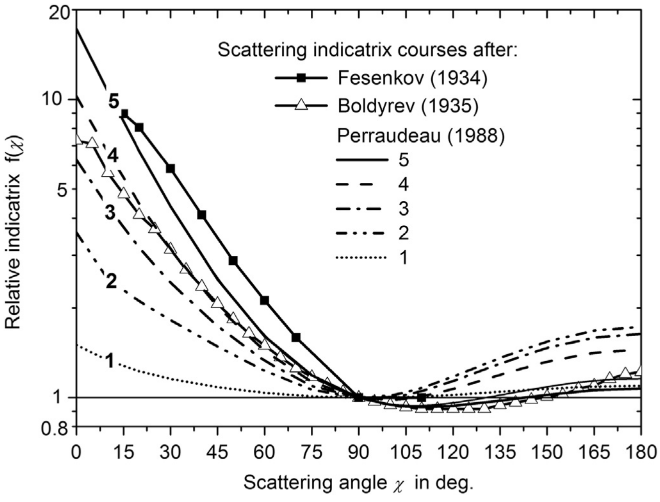

Fig. 3. Comparison of indicatrix courses of Russian indicatrices and after Perraudeau's five sky luminance distribution model.

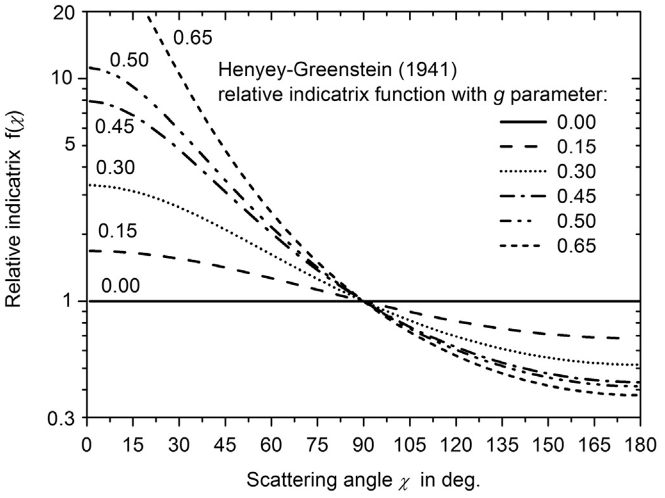

Fig. 4. Henyey and Greenstein indicatrix functions suggested for polluted atmospheres.

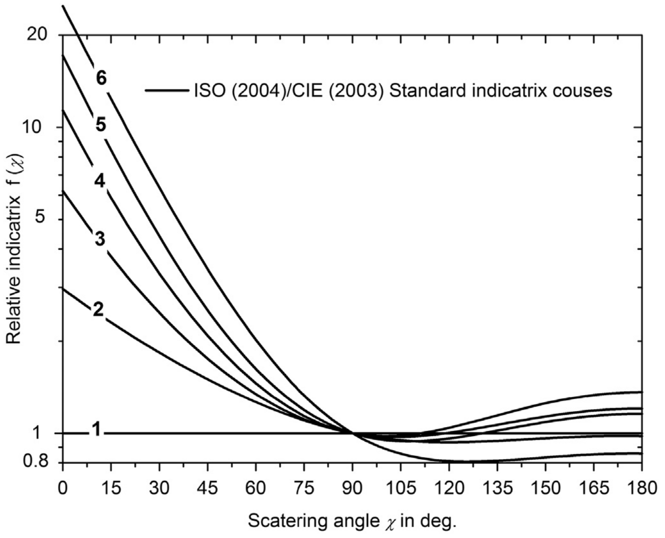

Fig. 5. Currently standardised six indicatrix functions by ISO and CIE.

without proof of the relationship between molecular and aerosol content or measured turbidity. Furthermore, Henyey–Greenstein indicatrix courses were derived from environments outside the Earth's atmosphere, so it seems that the extreme tail fall is questionable.

The new developments of more sophisticated objective instruments delayed precise sky luminance measurements until the spectral and cosine correction of photo-sensors were made by Dresler [19] and Barnard [20] and finally after 1982 the first computer-aided sky scanners were available for quick sky luminance records. Therefore, at the end of the 20th Century regular sky scans were reported and further studies were possible. Finally the CIE and ISO agreed on the standardisation of fifteen typical sky types in the whole range from overcast to clear sky types [21] including six standard gradation functions with courses in Fig. 1 and six indicatrix functions with courses in Fig. 5.

4. Possible concepts and conditions characterising sky luminance patterns

The basic concepts to determine available solar energy transformation to either radiance or luminance distribution on the sky vault or its irradiance or illuminance of horizontal surfaces on the ground level can follow two routes:

1. Due to rare local observations measured directly in photometric units by sophisticated instruments calibrated after human spectral visual photometric sensitivity $V(\lambda)$, e.g. by luminance scanners or zenith luminance metres and direct solar illuminance trackers, often instead of regular recordings only individual measurements are made of global and diffuse horizontal illuminance or vertical once by four sensors oriented to cardinal points. After the initiative of CIE regular data where gathered from 1994 in the IDMP general stations, also in longer terms, typical mean or extreme cases can be selected and studied in their minute, daily courses or seasonal variations of local climate characteristic frequency of occurrence or availability trends. Additionally sky luminance scans could be analysed to separate gradation and scattering courses as fundamental influences depending on sky luminance patterns.
2. To utilise world-wide regular long term data gathered by meteorological network recording global $E_{e,g}$ and diffuse $E_{e,d}$ horizontal irradiance data with auxiliary observations of relative sunshine duration, cloud cover and type, water vapour and pollution content of the atmosphere. These observations could be taken either from various locations or from satellite continental measurements, characterising regions or countries anywhere. However, these data are valid for the broadest radiation spectrum, thus when applied to daylight purposes the only possibility is to transform these data using questionable luminous efficacy approximations. Furthermore horizontal irradiance levels could result from various sky radiance distributions, hence, sky luminance patterns extracted from these may be inaccurate. Therefore the auxiliary overall conditions determining sky cloudiness state have usually to be taken first.

The first approach to take into account directly only the evaluated influx in the visible spectrum of extraterrestrial sunlight specific for daylight science and evaluation methods for skylight on exterior house fronts or within interior spaces. Thus sky luminance distributions could be specified with individually measured or typical cases determined on more precise data to define their integration and analysis of prevailing gradation and indicatrix courses for typical sky luminance patterns resulting in illuminance levels outdoors or in interiors.

The second approach based on ground or satellite data has the advantage of world-wide application possibilities specified on local conditions linking also the radiation and daylight similarities and long-term time detection of average hourly, daily, monthly, seasonal or yearly availabilities.

4.1. Modelled luminance scans and indicatrix for sky type classification

After the first clear sky luminance patterns proposed by Boldyrev before the second World War only a few research activities were tasked to discover the basic influences except the studies of atmospheric scattering indicatrix definitions by Sobolev [22] and Krat [23] during the war.

Although Sobolev [24] mentioned different types of indicatrices, e.g. binomial, ellipsoidal and the Rayleigh as well as Henyey and Greenstein types, there is no indication of indicatrix changes with the turbidity and cloud content except a note on overcast sky conditions when the sunlight transmission was radically reduced.

However, Krat's attempt to determine an indicatrix exponential function formula is simple and excellent because it is derived for the set of arbitrary homogeneous atmospheric conditions as

$$f(\chi)=1+N\left[\exp(-3arc\chi)-0.009\right]+M\cos^{2}\chi \qquad (11)$$

After his measurements in Tashkent when he tried to determine the turbidity and cloudiness influence after parameters:

1. N found to be close to 0 under overcast skies, while roughly between 8 and 12 under clear skies and even to 17 under cloudless polluted turbid skies,
2. M was expressing the raised indicatrix tail by values from 0 to 0.5,
3. χ is the angular distance from the direction of the original parallel sunbeam with zenith angle Z_s to any arbitrary sky element on the sky vault which has the zenith angle Z and azimuth angle A_z after the spherical geometry formula:

$$\chi=arc\ \cos(\cos Z_s\ \cos Z+\ \sin Z_s\ \sin Z\ \cos A_z) \qquad (12)$$

When the first author of this paper was asked to propose to the CIE Daylight Expert Committee the clear sky luminance formula, after further studies he proposed a modified, more general indicatrix expression for clean countryside localities [25]

$$f(\chi)=0.91+10\left[\exp(-3arc\chi)\right]+0.45\cos^{2}\chi \qquad (13)$$

During the first comments and discussions among committee members a proposal by Gusev [26] was accepted to add also more turbid and polluted conditions in town and urban areas after:

$$f(\chi)=0.856+16\left[\exp(-3arc\chi)\right]+0.3\cos^{2}\chi \qquad (14)$$

and so two CIE Clear Sky relative sky patterns were adopted by CIE [27].

Further studies by Kittler [28] explained the influence of atmospheric turbidity in homogeneous skies after the luminous turbidity factor T_v on the changes of both auxiliary parameters N and M values in the indicatrix formula Eq. (11), if T_v is in the range 1.5–72, thus

$$f(\chi)=1+4.3T_v\exp(-0.35T_v)\left[\exp(-3arc\chi)-0.009\right]+0.71T_v\cos^{2}\chi \qquad (15)$$

So, the indicatrix shape is determined after the quasi-homogeneous turbidity state of the atmosphere and also with its thickness and light transmission. A considerable step in progress had the first longer term regular sky scan measurement data recorded by the Lawrence Berkeley Laboratory team in San Francisco during 1985–1986 [29]. After these homogeneous atmospheric conditions data could be found within selected days when scattering indicatrix changes were analysed and the validity of a general indicatrix formula was tested and approved for 15 sky types [30] as

$$f(\chi)=1+c\left[\exp(-darc\chi)-\exp(d\pi/2)\right]+e\cos^{2}\chi \qquad (16)$$

At the same time within the Slovak–U.S.A. grant project [31] the gradation function changes were analysed from sky scans after a method explained in the book by Kittler et al. [7, pp. 115–118], and a generally valid formula was recommended after

$$\varphi(Z)=1+a\exp(b/\ \cos\ Z) \qquad (17)$$

with a form suitable after its normalisation to unity zenith value as

$$\varphi(Z)=\frac{1+a\exp(b/\ \cos\ Z)}{1+a\exp b} \qquad (18)$$

The previous Moon and Spencer overcast sky standard in cosine form was slightly changed to the general exponential type as shown in Fig. 1, where also the Boldyrev's tabulated values are inserted after their normalisation to zenith unity.

It is to be noted that in the ISO/CIE [21] definition of gradation and indicatrix parameters the following influences are taken into account:

1. a determines the extent to which horizon is either darker (plus values) or brighter (minus values) than zenith;
2. b indicates the influence of the air mass more dense close to the horizon by minus values;
3. c determines the luminance rise towards the sun position;
4. d expresses the gradient of the circumsolar halo;
5. e indicates the horizon luminance changes due to solar angular distance.

These standards use six combinations of indicatrix and gradation functions that were found as typical after selected sky scan analysis as well as selected for $L_{vZ}/E_{v,d}$ ratios. Due to the mathematical definitions of both functions they are easily applied in computer programmes to produce graphs of plan projected hemispheres with relative luminance isolines. The ISO/CIE standards defining homogeneous 15 sky luminance patterns including the slightly modified former CIE Overcast Sky, Lambertian unity sky and the CIE Clear Sky with both indicatrices in relative terms. These can serve as good practical solutions, which also facilitate determination of luminance distributions on the whole sky vault in physical units, i.e. in cd/m^2 (see [30,31]) if needed. In accordance with the atmospheric turbidity content there is a relation between sunlight and skylight proportion [32]. Furthermore, also patches of sky patterns can be determined within solid angles of windows and interior illuminance distributions can be calculated after Roy's computer programme [33].

4.2. Methods determining sky luminance patterns after measured irradiance data

The second system of "all weather" sky types extracting the information from available horizontal global and diffuse irradiance was proposed by Perraudeau [34], who grouped various sky conditions into five distinct categories (Table 1) using the classification after his Nebulosity Index I_N

$$I_N=\frac{1-CR_M}{1-CR_T}=\frac{1-E_{e,d}/E_{e,g}}{1-0.12037\ \sin\ \gamma_s^{-0.82}} \qquad (19)$$

where CR_M is the Cloud Ratio equal to the ratio of measured diffuse to global horizontal irradiance,

CR_T is the same diffuse to global horizontal irradiance ratio under an absolutely clear sky taken after Perrin de Brichambaut [35].

Perraudeau avoided any other classifying Index except his I_N and introduced instead of the ratios $L_{\chi Z}/L_{vZ}$ or $L_{vZ}/E_{v,d}$ his own

Table 1
Perraudeau's five sky types with auxiliary parameters a_1–c_1, a_2 and b_2 and a_3–c_3.

Sky type	I_N range	a_1	b_1	c_1	a_2	b_2	a_3	b_3	c_3
Overcast	0–0.05	32.33	13.16	3.24	1.18	0.23	0.76	0.13	0.29
Intermediate overcast	0.05–0.2	17.28	23.99	13.35	1.70	0.89	0.45	0.10	0.59
Intermediate mean	0.2–0.7	14.41	69.70	10.18	2.03	1.31	0.83	−0.29	0.38
Intermediate blue sky	0.7–0.9	13.05	124.96	7.49	2.21	1.54	0.83	−0.28	0.42
Blue sky	Over 0.9	12.89	243.38	3.26	2.25	1.59	0.83	−0.41	0.20

irradiance based $L_{vZ}/E_{e,d}$ which was quite unusual and curious because of the mix of luminance divided by irradiance units after

$$\frac{L_{vZ}}{E_{e,d}} = f'(\chi) g'(Z) h'(Z_s) \tag{20}$$

where the functions simulating the scattering indicatrix and two mixed functions simulating sky gradation are calculated after auxiliary parameters in Table 1 after equations

$$f'(\chi) = a_1 + b_1 \exp(-3\chi) + c_1 \cos^2 \chi \tag{21}$$

$$g'(Z) = a_2 + b_2 (\cos Z)^{0.6} \tag{22}$$

$$h'(Z_s) = a_3 + b_3 \cos Z_s + c_3 \sin Z_s \tag{23}$$

It is interesting that Eq. (20) is similar to the $L_{vZ}/E_{v,d}$ classification ratio which is determined for homogeneous sky types used by Kittler et al. [7,28] in different ways. So the differences can be tested and compared with $L_{vZ}/E_{e,d}$ and also a true normalised relative indicatrix could be calculated from $f'_n(\chi)$ to be inserted in Fig. 3 after the approximation

$$f'_n(\chi) = \frac{a_1 + b_1 \exp(-3\chi) + c_1 \cos^2 \chi}{a_1 + 0.0089833 b_1} \tag{24}$$

Another system in this category was published by Perez et al. [36] which serves to approximate momentarily occurring simulated sky patterns which correspond with the simultaneously measured diffuse and direct normal solar irradiance or diffuse and global irradiance assuming that the date Solar Constant and time of measurements determining solar altitude is recorded. This model is based on sophisticated sky scan measurements and theoretical studies which describe the overall “all weather” situation by two parameters:

1. The Sky Clearness $\in$ after

$$\in = \frac{(E_{e,d} + E_{e,ns})/E_{e,d} + 1.041 Z_s}{1 + 1.041 Z_s} \tag{25}$$

   where Z_s is the zenith angle of the sun in rad,
   $E_{e,nS}$ normal incident of direct solar irradiance
2. The Sky Brightness Δ which is similar to $\frac{E_{ed}}{E_o \sin \gamma_s}$ but instead of $\sin \gamma_s$ is used the optical air mass m is used after

$$\Delta = m \frac{E_{e,d}}{E_o} \tag{26}$$

Based on these two parameters using Perez's sub-parameters the gradation and indicatrix courses are also determined and correspond approximately with those by Kittler [25]. Another set of sub-parameters a_i, c_i, c'_i and d_i are proposed by Perez et al. [36] to calculate the zenith luminance L_{vZ} in kcd/m^2

$$L_{vZ} = E_{e,d} \left[a_i + c_i \cos Z_s + c'_i \exp(-3Z_s) + \Delta d_i \right] \tag{27}$$

Note, that Perez's indicatrix is not exactly in the normalised relative term and his system does not contain the first overcast ISO/CIE sky type due to its absence in San Francisco scan data. However, due to the USA government sponsorship of its authors this system is used in many simulation computer programmes, e.g. Radiance, EnergyPlus, ESP-r. Furthermore, this method in its application needs time and location dependent irradiance data either from local or satellite measurements.

Seven years later a third method based on irradiance data by Igawa et al. [37] was published and tried to reflect recent progress and new developments as

1. Acceptance of a new value of the Extraterrestrial Solar Constant $E_{eo} = 1366.1$ W/m^2 by Geymard [38].
2. The Luminous Solar Constant $E_{vo} = 133.8$ klx derived by Darula et al. [39].

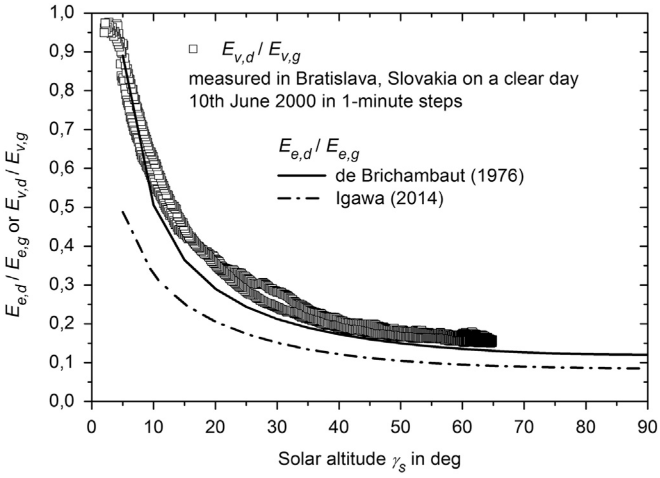

Fig. 6. Comparison of radiant and visible diffuse theoretical ratios to measured data.

3. Dependence of the on ground horizontal irradiance and illuminance levels on the extraterrestrial levels and scattering conditions within the atmospheric layers which create the radiance/luminance patterns on the sky vault.
4. Regular daily recordings in minute steps of daylight situations within the CIE IDMP programme with additional sky scan recordings producing data for analysis and comparison tests.
5. Studies of special, seasonal or long-term availability related to local cloudiness cover, cloud type, turbidity and pollution.

All these new research results enabled formulation of better fits to theory and reality or to improve previous methods, e.g. in case of the Igawa model [37,40].

Igawa's [37] method used Perraudeau's [34] Nebulosity Index in a renamed form as “Cloudless Index” *Cle*

$$Cle = \frac{1 - E_{e,d}/E_{e,g}}{1 - Ces} \tag{28}$$

where

$$Ces = 0.01299 + 0.07698\, m - 0.003857\, m^2 + 0.0001054\, m^3 + 0.00001031\, m^4 \tag{29}$$

while in the improved version Igawa [40] the same parameter is determined after solar altitude γ_s thus

$$Ces = 0.08302 + 0.5358 \exp(-17.3\gamma_s) + 0.3818 \exp(-3.2899\gamma_s) \tag{30}$$

In comparison with measurements recorded at Bratislava in the visible range, i.e. $E_{v,d}/E_{v,g}$ ratios gathered during an ideal clear day in Fig. 6 it is evident that Perraudeau's CR$_T$ applying de Brichambaut's relation was a wise choice of a simple and truthful interrelation dependent on the solar altitude while Igawa's ratio is a bit lower.

However, Perez's suggestion to add a second classifying parameter “Sky Brightness Index” renamed to “Clear Sky Index” was also adopted by Igawa and was rewritten to

$$Kc = \frac{E_{e,g}}{Seeg} \tag{31}$$

where *Seeg* is determined in W/m^2 and should represent a case of lowest atmospheric turbidity for Linke Turbidity Factor T_L equal to 2.5 and thus in the original paper Igawa [37] is

$$Seeg = \frac{0.845 E_{eo}}{m} \exp(-0.067\, m) \tag{32}$$

but later improved by Igawa [40] for T_L equal to 2:

$$Seeg = \frac{0.84 E_{eo}}{m} \exp(-0.054\, m) \tag{33}$$

However, if *Seeg* should represent the reduced horizontal direct solar irradiance on ground level then after originally Bouguer's formula the relation has to follow

$$E_{es} = E_{eo} \exp(-a_e m T_L) \sin \gamma_s \tag{34}$$

Finally in the original 1997 method Igawa [37] proposed the "Sky Index" *Si* for determining his set of twenty sky types after:

$$Si = Kc + Cle^{0.5} \tag{35}$$

which was improved in his 2014 paper only in the acronym as

$$Siv = Kc + Cle^{0.5} \tag{36}$$

It is also interesting that the indicatrix and gradation functions used were the same as in the ISO/CIE standards, i.e. after formulae (16) and (18) as well as the basic formula after Eq. (9) adopted in Igawa's improved version. Now every reader is probably curious why has Igawa introduced so many indexes and auxiliary parameters when after *Si* he also investigated Perraudeau's system of five sky types from overcast with *Si* smaller than 0.3 to almost overcast ($Si = 0.3$–0.6), mean ($Si = 0.6$–1.5), nearly clear ($Si = 1.5$–1.7) to absolutely clear when *Si* is over 1.7. However, his final system of 20 sky types is based on the whole range of *Si* values from 0.1 to 2 divided by 0.1 steps respecting the ISO/CIE standards fully. Thus it seems that the whole system of irradiance and illuminance patterns as well as both approaches to their determination is linked with the resulting irradiance and illuminance horizontal levels.

Contrary to Perraudeau's one parameter categorisation system both Perez and Igawa normalise sky types by a second parameter called either Sky Brightness after Eq. (26) or Clear Sky Index after Eq. (31) in irradiance terms or also Normalised Global illuminance by Igawa et al. [42] as

$$E_{vgm} = \frac{m E_{v,g}}{E_{vo}} \tag{37}$$

and "Relative Global Illuminance" in Igawa and Nakamura [43]

$$N_{vgm} = \frac{E_{v,g}}{Sevg} \tag{38}$$

where

$$Sevg = -36.78\gamma_s^5 + 188.79\gamma_s^4 - 375.95\gamma_s^3 + 306.23\gamma_s^2 + 15.47\gamma_s + 0.83 \quad (\text{klx}) \tag{39}$$

However, these last two parameters are not mentioned in Igawa's improved method.

In Kittler et al. books [7,41,44] and earlier papers both global and diffuse horizontal irradiance or illuminance were normalised by extraterrestrial horizontal ones respectively as ratios:

$$\frac{E_{e,g}}{E_{eo,h}} = \frac{E_{e,g}}{E_{eo}\ \sin\ \gamma_s} \quad \text{and} \quad \frac{E_{e,d}}{E_{eo,h}} = \frac{E_{e,d}}{E_{eo}\ \sin\ \gamma_s} \tag{40}$$

or

$$\frac{E_{v,g}}{E_{vo,h}} = \frac{E_{v,g}}{E_{vo}\ \sin\ \gamma_s} \quad \text{and} \quad \frac{E_{v,d}}{E_{vo,h}} = \frac{E_{v,d}}{E_{vo}\ \sin\ \gamma_s} \tag{41}$$

All these parameters are compared in Fig. 6, in the ratio $\frac{E_{e,d}}{E_{e,g}}$ or $\frac{E_{v,d}}{E_{v,g}}$. The problem of the difference in using $\frac{m}{E_{eo}}$ or $\frac{m}{E_{vo}}$ contrary to E_{eo} sin γ_s or E_{vo} sin γ_s is evident as no extraterrestrial horizontal irradiance or illuminance can be influenced by the relative air mass of the atmosphere and, furthermore these extraterrestrial solar constants defined on fictitious planes in normal position to sunbeams have to obey the cosine law of incident beams which in case of the fictitious horizontal plane is equal to the sine of the solar altitude.

5. Possibilities to detect the relative scattering indicatrix and gradation functions from all-sky scan data

Studies to measure the luminance distribution on arbitrary real sky vaults were enabled with a new Pacific Northwest laboratory scanner that started by Karayel and the Berkeley Lab team [29] recording regularly measured data from June 1985 to December 1986. However, nobody tried to analyse these data until the method for deriving the scattering indicatrix [45] and the gradation function was published (both in the book by Kittler et al. [7, pp. 115–118]). From the roughly recorded 16,000 measured scans Richard Perez, the co-author in the Slovak-U.S.A. grant project, selected 88 quasi-homogeneous all-sky scans to be analysed and could serve for the practical set of fifteen sky luminance patterns in the range from densely overcast to cloudless skies. Furthermore some available sky scans measured at Sydney airport [46] and in Tokyo [47] were analysed for comparison reasons. However, from the private research report [48] only the analysis of the Berkeley scan 164/85 could be placed in the book (Figs. A4.4 and A4.7 in [7]), which is the example of a very clear cloudless sky with very low turbidity. To show comparison of other indicatrix functions are shown in Fig. 7a using the scan results after measurements in Sydney on a cloudless clear day while evaluated relative indicatrices trends are in Fig. 7b. Both scan data and indicatrix courses are compared to similarly analysed Tokyo scans in Fig. 8a and b measured under roughly the same clear sky conditions.

It is evident that besides some influences caused by the different sky scanners used (measurements in Sydney by the German

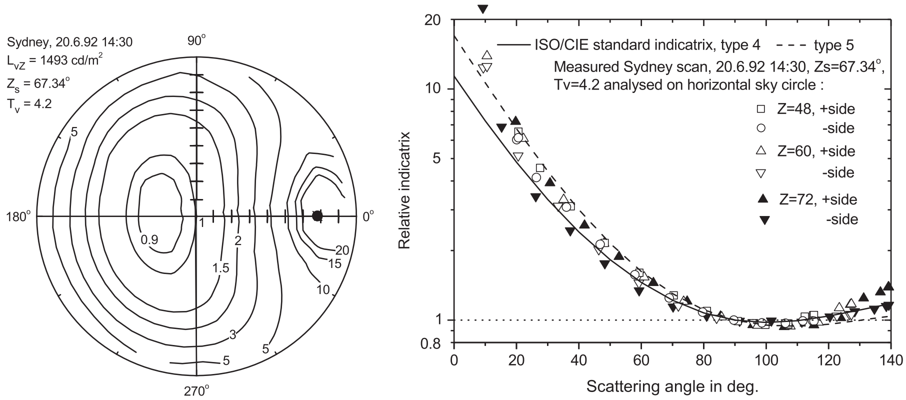

Fig. 7. The luminance distribution map and evaluated indicatrix courses after measurement in Sydney.

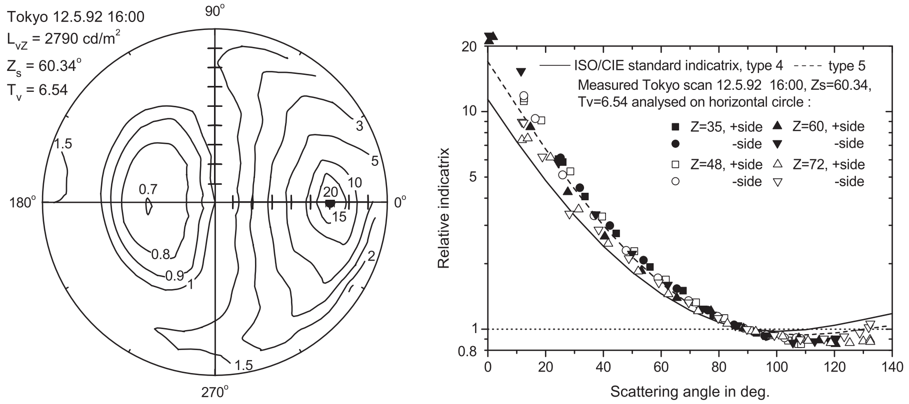

Fig. 8. The luminance map and evaluated indicatrix courses after measurement in Tokyo.

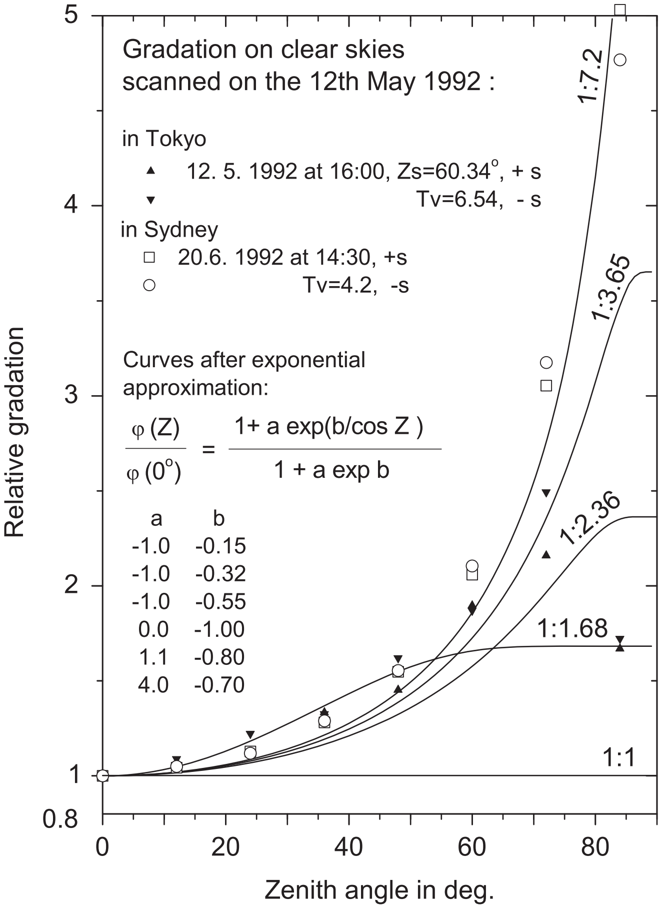

Fig. 9. Relative gradation measured from Tokyo and Sydney scans.

PRC Krochmann scanner while Tokyo scans were measured by the Japanese EKO scanner) and regarding different local turbidity situations both indicatrix and sky luminance maps are alike showing the high luminances concentrated around the sun position with the lowest luminances at 90–100° distance from the sun on the solar meridian corresponding with the indicatrix course trends.

However, note that both the PRC Krochmann and EKO scanners have relatively wide acceptance angles 7.5° and 11° respectively both following the Tregenza sky subdivision [49] which for defining the indicatrix and gradation measurements are insufficiently accurate [50]. Luminance metres with the 1° acceptance angle could be used to get closer to the solar disc rim but still avoid the extreme solar luminance which is approximately 700×10^6 kcd/m^2 and either endanger the luminance metre or distort the indicatrix measurements resulting in an apparent enormous rise of the forward scatter.

Gradation function analysis is in both cases documented in Fig. 9 with an increasing tendency from zenith to horizon indicating a cleaner atmosphere in Sydney and corresponding to a quite polluted sky closer to horizon in Tokyo. Due to the very high turbidity close to horizon a significant drop in measured luminance values from relative gradation roughly 1:3.65 to 1:1.68 is evident.

6. Results and discussion

Since 1993 several researchers [51–53] have analysed indicatrix and gradation courses from sky luminance scans measured in France near Lyon, in England, at Watford near London or in Brazil at Itatiba near Sao Paulo. Recently CIE published a special guide for practical users with information about the application and use of the ISO/CIE Standard [54]. All the results have verified the ISO/CIE standard indicatrix and gradation function or used these courses as reference standards for comparisons. Especially evident is the fact that under a clear sky the scattering influence of aerosol turbidity at first elongate the indicatrix forward with a minimum at scattering angles 90–105°, then slowly rise again in a backward tail. However, with the increasing density of the aerosol environmental content a gradual approach to a uniform unity course of the relative indicatrix function is evident. Some older formulae, e.g. Eqs. (8) or (10) ignored these course trends especially the air molecule influences seemingly exaggerated by Rayleigh's indicatrix after Eq. (7) in the absolutely unpolluted atmosphere.

It is unfortunate that there are no sky tailored tools to determine precisely the scattering indicatrix courses measured along the sky almucantars and the gradation courses along the sky section circle from zenith to horizon with the constant angular distance of the sun position to zenith. Such research scanners could identify also the placement of cloud patches and their influence on scattered skylight.

Even more precise information on real momentary sky luminance patterns could yield photo images taken by digital cameras with a fisheye lens reproducing the luminance distribution on the whole sky vault [55]. The advantage of a photo image of any arbitrary real sky luminance pattern whether close to homogeneous or any non-homogeneous distribution is in its instant cloudiness picture, which depicts the momentary placement, distribution cover of momentary cloud groups as well as cloud types present, [56]. However, to analyse such a sky image the calibration of the fisheye lens projection system is needed and the calibration of luminances in physical units for different pixels is to be mathematically expressed, e.g. in [57].

It seems that the range of almost "all weather" ISO/CIE standardised sky types is covered. However, it is evident that only homogeneous skies do not represent also the multiple quasi- or non-homogeneous sky cases especially these occurring under differently distributed clouds and cloud masses of various cloud cover and mixture of cloud types that can prevail within differently oriented solid angles of building apertures or photovoltaic panels. Their number and frequency can influence the overall harvest and seasonal availability of the efficiently available energy or luminous flux.

It is true that for instance especially in the varied luminous climate of Central Europe after the analysed 5-year data gathered during 1994–1998 in Bratislava, that the strict selection of homogeneous cases was only about 46% of the overall number of cases measured in 5-min intervals [58–60].

Thus future fundamental research is expected to analyse and cover nonhomogeneous sky luminance patterns as indicated in a recently published trial on the various approaches and problems [58].

7. Conclusions

It is remarkable that historically research and development gradually evolved to understand the main influences necessary for determining the sky radiation and illumination effects. The interrelated systems of corresponding radiation and photometric terms and units, measurement instruments and uncoordinated research by individuals or teams in astronomy and meteorology, radiation and daylight science led to uncovering possibilities for solar energy use in computer based and aided engineering design and utilisation of natural daytime radiation and lighting conditions.

The atmospheric transmission and scattering of originally parallel sunbeams by cloudiness and turbidity created direct and diffuse sky radiation and daylight on ground level with a multiple variety of sky radiance and luminance patterns determining ever changing daytime environmental conditions to be utilised by human senses and habituation. Gradually the knowledge and laws defining these multiple influences predetermining human life and everyday wellbeing, work and social activities were found.

The countless number of regular irradiance and illuminance recordings of data gathered by the net of stations world-wide as well as satellites give momentary cases that can be analysed and specified (e.g. as shown in the Perez method), or can be summarised in specific groups (e.g. after Perraudeau or Igawa) using less or more complex systems of parametrisation. However, all models based on global/direct and diffuse irradiance data inherently represent the whole broad solar spectrum so have to be corrected by approximate luminous efficacies for the human visual sensibility spectrum to fit objective photometry. The CIE IDMP stations since 1992 are regularly recording global and diffuse sky illuminance with either zenith luminance or sky luminance scans, so these data can be now also analysed in more details.

The physically and mathematically determined quasi-homogeneous atmospheric situations especially under cloudless and overcast skies has resulted in the simple set that is now standardised by ISO and CIE in fifteen basic reference sky types ready for practical applications in architectural designs and further research.

The trend for the general description of naturally available sources for engineering applications has to be considered in the chain of natural events with items like – spectral solar radiation and light availability⇨extraterrestrially available irradiance or illuminance constants (E_{eo} and E_{vo})⇨ distortions in the atmospheric environment (transmittance, scattering, cloudiness, turbidity)⇨sky radiance/luminance patterns⇨horizontal sun and sky irradiance/illuminance at ground level⇨vertical sun and sky irradiance/illuminance on house fronts, windows, solar or photovoltaic panels⇨interior sun and sky irradiance and illuminance by patches of sky patterns⇨interreflection conditions due to interior surfaces.

Of course, any item or element in this interrelated chain could be investigated separately and might uncover elements of new information that could lead to better understanding of interrelations over the whole chain, or uncover some hidden effects in the complex mixture of influences.

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