Summary Daylight in buildings can save energy by reducing artificial lighting energy consumption. An accurate estimate of internal daylight availability is required for energy use predictions and to include the effect of window orientation this must be based on an adequate model of sky luminance distribution. For such energy calculations the 'average sky' (based on measurements for a wide range of real skies) is recommended as a replacement for the CIE overcast sky. This paper analyses luminance data obtained by Wegner to show that the luminance distribution of an average sky is of the form $L = ae^{-by} + c$ where y is the angle in degrees between the sun and the element of sky under consideration, b is approximately 0.025 and a and c are functions of solar altitude alone, and, significantly, do not depend on the altitude of the element of sky. Using this formula together with a direct solar component the paper derives expressions for horizontal and vertical illuminances, and the average illuminance inside a side-lit room; and briefly outlines further work, under way at BRS, to evaluate the model for weather conditions in the UK and to apply it to problems of daylighting design.

The luminance distribution of an average sky

PAUL J. LITTLEFAIR

1 Introduction

One of the ways energy can be saved in buildings is through better use of daylight. Recent research at BRE has concentrated on three important aspects of this: predicting the energy savings due to photoelectric controls[1,2], predicting artificial lighting use[3], and optimising window size[4]. However, any attempt to quantify the energy saving effects of daylight will require an accurate estimate of the amount of daylight available. This includes not just the total amount of light coming from the sky (usually represented by the horizontal illuminance outdoors), but also the way that light is distributed over the sky vault. This luminance distribution is important because a point in a room, for instance, will only receive light from one particular part of the sky.

The CIE overcast sky gives the idealised luminance distribution for one particular type of weather condition. It is useful for 'worst-case' design studies of daylighting in buildings, but its use for energy related calculations is inappropriate because these will need to include the effects of overcast, clear and partially clouded skies, i.e. all weather conditions. A mathematical model produced at BRE by Loudon (to standardise radiation work[5]) and later modified by Lynes and Crisp[6] provides a more realistic form of luminance distribution to represent an average over a wide range of weather conditions. Among other things this model of sky luminance can predict the effect of the orientation of the window walls in a building—an important consideration ignored by present daylighting design practice.

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This paper develops the BRE model making use of luminance data measured by Wegner[7,8]. It shows how a simple extension of the BRE model can be used to accurately predict the average luminance distribution of a real sky and its variation with solar altitude. The formula obtained is used to predict the illuminance values on unobstructed horizontal and vertical planes.

The work described forms part of a programme of investigation aimed at providing practical daylight design methods for predicting internal illuminances on a more realistic basis than current practice allows. Measurements of daylight availability on vertical as well as horizontal surfaces are required to adapt the model to UK conditions (Wegner's measurements were made in Berlin) and these are being carried out at BRE. Simultaneous measurement of internal illuminances under real skies are being made and will be used to investigate how the theory can be employed by designers for prediction of interior daylight conditions.

2 The BRE daylight model

Fig. 1 shows the basis behind the BRE model. Light from the sun and sky is divided into three components:

(1) A 'direct solar' component which comes directly from the sun;

(2) A 'circumsolar' component which comes from a region around the sun;

(3) A 'background diffuse' component, which is assumed to be uniformly distributed over the whole sky.

Lynes and Crisp justified this last assumption by considering an 'average sky' as the mean of a succession of clear and overcast skies[6], a technique used more recently by Aydinli[9]. A clear blue sky has a bright horizon and a darker zenith, while a

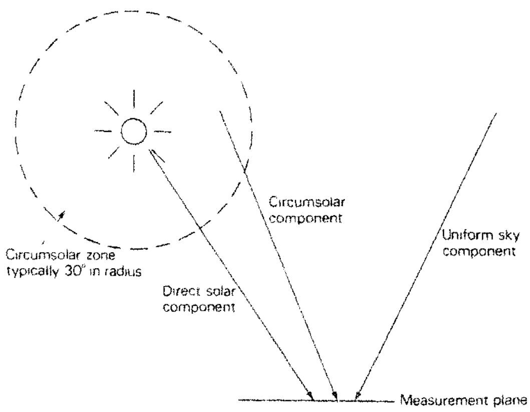

Fig. 1. The BRE daylight model

fully overcast sky has a bright zenith and a darker horizon. So a succession of clear and overcast skies might, on average, have a roughly uniform luminance distribution.

For components (1) and (3) it is quite easy to calculate the illuminance on a horizontal or vertical plane from the magnitude of the component itself. Let S be the normal illuminance due to the direct solar component in lux, and U the luminance of the background diffuse component, in cd/m^2.

Then for a horizontal plane

illuminance due to the direct solar component $= S \cos \theta_s$
where θ_s is the zenith angle of the sun ($\theta_s \leqslant 90°$) (Fig. 2) (1)
illuminance due to the background diffuse component $= \pi U$ (2)

and for a vertical plane

direct solar illuminance $= S \sin \theta_s \cos \phi_{REL}$ (3)
where ϕ_{REL} is the azimuthal angle between the sun and the normal to the vertical plane $|\phi_{REL}| \leqslant 90°$
background diffuse illuminance $= \frac{1}{2} \pi U$ (4)
regardless of the orientation of the vertical plane.

Formulae for the solar altitude and azimuth at a particular day and time are given by Petherbridge[10] and also by Lynes and Crisp[6].

It is more difficult to calculate the effects that the circumsolar component will have on horizontal and vertical illuminance. The circumsolar zone can vary not just in mean luminance, but also in angular radius, and in the luminance distribution within the zone. Lynes and Crisp[6] overcame this problem by assuming that the circumsolar component could be amalgamated with the other two components. Light coming from a small region around the sun, of angular radius less than 30°, could be treated as coming directly from the sun. The remainder (if any) of the circumsolar light could be added to the background diffuse component. Thus Equations 1-4 would then read

$$E_H = (S + \beta_H C_s) \cos \theta_s + \pi(U + C_u) \quad (5)$$

$$E_V = (S + \beta_V C_s) \sin \theta_s \cos \phi_{REL} + \frac{\pi}{2} (U + C_u) \quad (6)$$

where C_s, C_u are the amounts of circumsolar light added to the two components. β_H and β_V are functions of solar altitude and azimuth and correct for cases where a plane does not receive light from all the circumsolar zone (for a horizontal plane this would be when the sun was low on the horizon). Where a plane receives light from all the circumsolar zone, β equals 1.

Equations (5) and (6), therefore, set the luminance of an average sky to a constant apart from a small region around the sun. But is this really a valid assumption to make? Although Crisp and Lynes[6] were able to fit horizontal illuminance data to this simple model, values of horizontal illuminance alone are not enough as any sky luminance distribution (for example a point source at the zenith) can be fitted to horizontal illuminance data. Crisp and Lynes suggested the use of vertical illuminance data (as yet unobtained) to provide a more critical test of the model. Alternatively, what is required is a large number of luminance measurements of different parts of the sky, averaged over a succession of real skies. Fortunately in 1968/9 such measurements were made, by J. Wegner at the Technical University of Berlin.

3 Wegner's experiment

Wegner[7, 8] measured the luminance of the whole sky over a year. His apparatus resembled a telescope; an ingenious swivelling mechanism allowed it to scan the whole sky in 30 minutes. Readings began before sunrise and ended after sunset.

The data are presented in tables, one for each solar altitude h_s in 5° steps from 0° to 60°. The tables give the average luminance of a patch of sky of particular altitude h, and azimuth α relative to the sun. The intervals in h and α are 6° and 12° respectively. Over the whole year some seven million readings were made; so most of the luminance values are the average of over two thousand readings.

The main causes of error in each reading were fatigue after the cell had been exposed to high light levels and poor colour correction; Wegner quotes five per cent for the error due to each of these. Other errors arose because the meter could not record the very high luminances close to the sun, and because the weather in the year selected was not truly representative of a longer period of climatic conditions. This last error, according to Wegner, was quite small. A fifth error was due to scattered sunlight entering the apparatus, but Wegner does not quote a value for this. All these errors are discussed in reference 7.

However, because each figure in the tables represents the average of a large number of readings, its absolute error will probably be somewhat less than ten per cent. It is hard to calculate exactly how large it will be because the errors are systematic.

The tables of data are Tables 5a to 5m of Wegner's thesis[7]. There is an English translation of this at BRS. A summary of Wegner's work is given in reference 8, an English copy of which resides in the BRS Library.

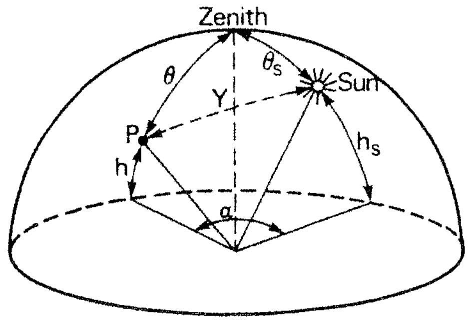

Fig. 2. Angles used in the calculation

4 Analysis of the luminance data

Wegner's tables gave the luminance of a patch of sky of altitude h and azimuth relative to the sun α, for a particular solar altitude h_s (see Fig. 2). To investigate the BRE model, however, we need to know another angle—the angle y between the patch of sky and the centre of the sun. This can be calculated from the formula

$$\cos y = \cos h_s \cos h \cos \alpha + \sin h_s \sin h \quad (7)$$

which is obtained using spherical trigonometry[11].

For a selection of solar altitudes, Wegner's luminance data were plotted against angle y. The sky luminance appeared to depend only on y for each particular solar altitude; no dependence on the altitude of the patch of sky h was observed. The only exception to this was for h = 6° (the lowest value) when ground reflected light affected the luminance values.

Moreover, the sky luminance appeared to fall off exponentially as y increased. There was no particular cut-off angle above which the sky luminance became uniform.

A computer program was written to fit an exponential curve to the data. It used a least squares technique to fit a curve of the form

$$L = ae^{-by} + c \quad (8)$$

to the data. All of Wegner's data points were used, apart from those for which h = 6° (see above), and for points very close to the sun. For these cases the cell was exposed to very high direct solar luminances which either produced high readings, or, because of the cell cut-off, inordinately low readings. This effect was worst for high solar altitudes, so the curve-fitting routine ignored points for which $y° < (5 + h_s°/4)$. This value was selected by inspection of the preliminary plots described earlier.

Using this program it was possible to fit a curve of the form shown in Equation (8) to give an RMS error in luminance of better than five per cent for most of the solar altitudes. Note that the right hand side of Equation (8) represents two components: a uniform sky component (c) and a circumsolar component (ae^{-by}).

4.1 Values of a, b and c

The best values of b (y measured in degrees) for each solar altitude are given in Table 1.

Table 1. Best values of b for each solar altitude h_s

$h_s°$	0	5	10	15	20	25	30
b_{opt}	0.041	0.033	0.032	0.029	0.028	0.026	0.026
$h_s°$	35	40	45	50	55	60	
b_{opt}	0.0245	0.0235	0.020	0.019	0.018	0.017	

The optimum b varies with solar altitude. As h_s increases b goes down, indicating a less steep fall-off of circumsolar luminance with angle from the sun y. This could be due to systematic seasonal effects; for example higher solar altitudes would be observed in the summer when less clouds would be present. This in turn could give rise to less attenuation of the circumsolar luminance over the sky vault.

Despite this variation in the optimum values of b, it makes only a small difference to the average error in L if b is set to 0.025 for all solar altitudes (see Table 2). A constant b simplifies illuminance calculations (in Section 5) considerably, so b was set to 0.025.

For b = 0.025 the best values of a and c for each solar altitude are plotted in Figs. 3 and 4. a and c both increase as solar altitude increases, as might be expected; but a tends to a constant for large solar altitudes. Empirical functions for a and c were found to be

$$a = 0.0456\, h_s^2\, e^{-h_s/30} + 0.27 \qquad \text{kcd/m}^2 \quad (9)$$

$$c = 0.2 + 0.1\, h_s - 0.18 \sin (10\, h_s) \quad \text{kcd/m}^2 \quad (10)$$

h_s in each case being measured in degrees. It should be emphasised that these formulae have no theoretical basis; in particular the 'sin (10 h_s)' term in Equation (10) is inserted simply to provide conveniently a better fit for c at low solar altitudes. We can combine Equations (8), (9) and (10) to give a complete empirical formula for average sky luminance

$$L = (0.0456\, h_s^2 e^{-h_s/30} + 0.27)\, e^{-y/40} + 0.2 + 0.1\, h_s - 0.18 \sin (10\, h_s) \quad (11)$$

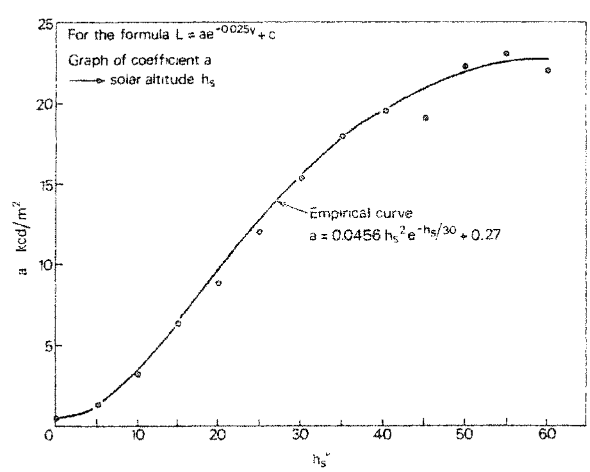

Fig. 3. Variation of a with solar altitude h_s

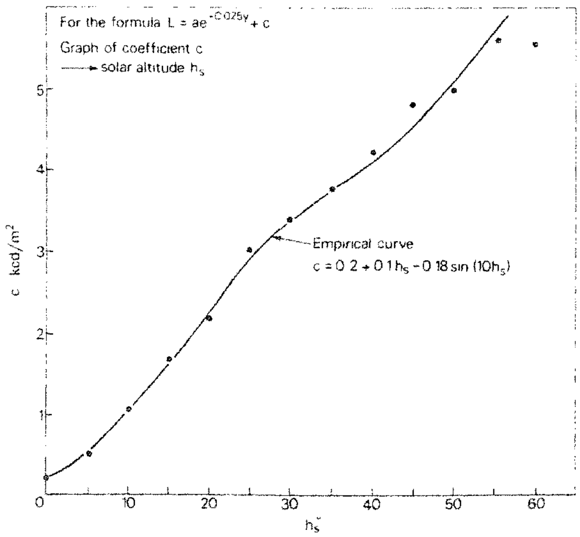

Fig. 4. Variation of c with solar altitude h_s

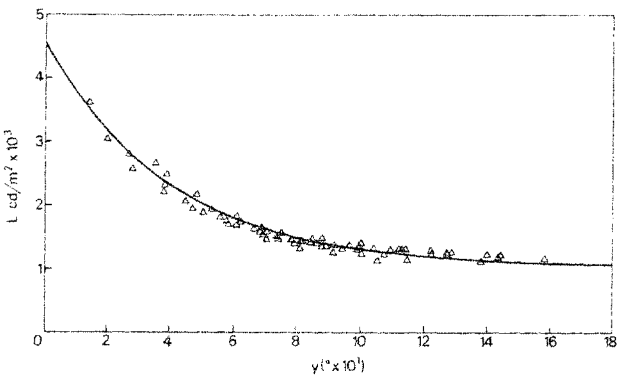

Fig. 5. Graph of L—y for solar altitude 10°

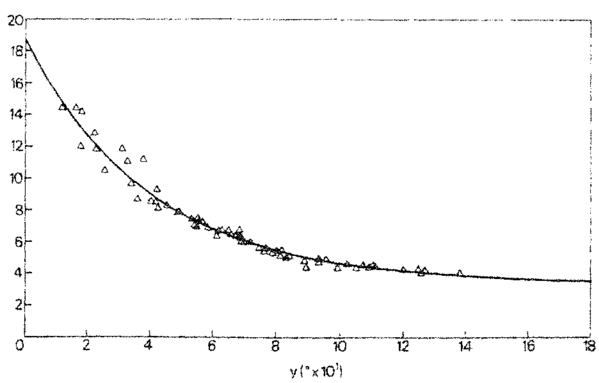

Fig. 6. Graph of L—y for solar altitude 30°

Figs. 5 to 7 are graphs of luminance against angle y for representative values of solar altitude h_s. One-third of Wegner's data points are represented by crosses. The curves in the graphs are those output by Equation (11).

The curves in general fit the luminance data very well, except for the extreme solar altitudes of 0° and 60°. One of the reasons for this is that atmospheric conditions and diffusive effects will vary with solar altitude, and so the value of b is not constant as assumed earlier. Nevertheless even for these solar altitudes the average (RMS) error in L is within 11 per cent.

Table 2 gives these errors for each solar altitude, and shows the effect each of the assumptions about a, b and c has on the errors in the fit to the luminance data. The errors quoted are standard deviations

(of the form $\sqrt{\left\{ \frac{1}{n-1} \sum \left(\frac{\text{Error in luminance}}{\text{luminance}} \right)^2 \right\}} \times 100$ per cent)

5 Predicting horizontal and vertical illuminances

One of the drawbacks of the empirical formula in Equation (11) is that it is based on data for Berlin rather than Britain, where the climate and solar position are different. Clearly measurements need to be made in Britain as well. However it is very difficult to make a large number of luminance measurements; it is easier to measure illuminance and an apparatus has been constructed at BRS to do this. Because of this, and because most daylight calculations are based on illuminance, it would be useful to find out how horizontal and vertical illuminance are related to solar altitude and azimuth and the formula for luminance distribution.

5.1 Horizontal illuminance

Equation (8) gives

$L = ae^{-by} + c$

and from this, with the addition of the direct solar component, we obtain

$$E_H = S \sin h_s + \pi c + a \int_0^{2\pi}\int_0^{\pi/2} e^{-by} \cos\theta \sin\theta \, d\theta \, d\phi \qquad (12)$$

where $\cos y = \cos h_s \cos h \cos \alpha + \sin h_s \sin h$

(Equation 7))

The double integral depends only on solar altitude and values for it are given in Table 3. The integral

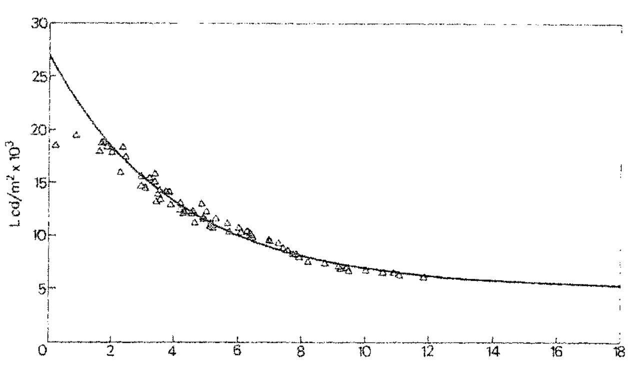

Fig. 7. Graph of L—y for solar altitude 50°

Table 2. Average errors in luminance values for the different assumptions

Solar altitude h_s (degrees)	Std deviation (per cent) for best-fit exponential curve a, b, c vary freely	Std deviation (per cent) for best curve with b = 0.025 a, c vary freely	Std deviation (per cent) for curve with b = 0.025 a, c given by equations
0	10.0	10.7	10.7
5	4.3	5.1	8.4
10	4.2	5.0	5.5
15	4.5	4.9	5.1
20	4.3	4.5	7.2
25	7.4	7.4	7.6
30	4.7	4.8	4.8
35	4.1	4.2	4.3
40	4.0	4.0	4.2
45	3.5	3.9	4.9
50	3.1	4.5	4.5
55	2.6	3.4	3.5
60	3.5	4.1	9.6

turns out to be almost exactly equal to $0.456 + 0.01\ h_s$ (h_s = solar altitude).

Thus $E_H = S \sin h_s + \pi c + aI$ (12a)

where $I \cong 0.456 + 0.01\ h_s$

and a, c are functions of h_s (see Equations (9), (10))

5.2 Vertical illuminances

These are more complicated to calculate because they depend not only on solar altitude but also on the azimuthal angle ϕ_{REL} between the sun and the normal to the plane. The relevant equation is

$$E_v = S \cos h_s \cos \phi_{REL}\ G|\phi_{REL}| + \frac{\pi c}{2} + a \int_{-\pi/2}^{\pi/2} \int_{0}^{\pi/2} e^{-by} \sin^2\theta \cos\phi\ d\theta\ d\phi \qquad (13)$$

($G|\phi_{REL}| = 1$ if $|\phi_{REL}| < 90°$
$= 0$ if $|\phi_{REL}| > 90°$

for if $|\phi_{REL}| > 90°$ the sun cannot shine on the plane and the solar component is zero).

A computer program was written to evaluate the double integral (J).

An approximate formula is

$$J = 0.21 + 0.31\ x + 0.19\ x^2 \qquad (14)$$

where $x = \frac{\sqrt{3}}{2} \cos h_s \cos \phi_{REL} + \frac{1}{2} \sin h_s$

but this is less accurate (five per cent error for some solar positions) than the corresponding equation for horizontal illuminance—so it is probably best to use Table 4. For computer work, J can be expressed as a Fourier series in ϕ_{REL}*. A suitable series is

$$1000\ J = (276 + 4h_s - 0.04h_s^2) + (257.5 + 4.17h_s - 0.087h_s^2) \cos(\phi_{REL}) + (45.4 + 0.89h_s - 0.087h_s^2) \cos(2\phi_{REL}) + (-2.3 - 0.05h_s + 0.001h_s^2) \cos(4\ \phi_{REL}) \qquad (15)$$

This formula is accurate to better than two per cent.

6 Further work

Since Wegner's measurements were made in Berlin validation of the luminance model should be carried out for British weather conditions. It seems reasonable that the basic form of the average sky luminance distribution will be similar. Measurements in different places indicate that for the extreme sky conditions, overcast[12] and clear[13, 14], the form of luminance distribution does not vary greatly with geographical position. Also Berlin and London have the same range of sky conditions (for each month of the year the average cloud cover in Berlin is within 10 per cent of that in London[15], and solar altitudes (only 1° difference in latitude).

However to provide a searching test for the luminance model under British conditions, an apparatus has been constructed at BRE to continuously monitor illuminance

(1) on an unobstructed horizontal plane;

(2) on four unobstructed vertical planes facing North, South, East and West;

(3) on a horizontal plane fitted with a guard ring to eliminate the direct solar component;

(4) at six points inside each of four model rooms facing North, South, East and West.

The results from this will be used to investigate the daylight model. From the readings (1) and (2) it is,

*J. A. Lynes, private communication

Table 3. Value of integral I for different solar altitudes

Solar height h_s°	0	5	10	15	20	25	30
Double integral I	0.456	0.503	0.552	0.604	0.657	0.711	0.765
Value of $0.456 + 0.01\ h_s$	0.456	0.506	0.556	0.606	0.656	0.706	0.756
Solar height h_s°	35	40	45	50	55	60	
Double integral I	0.818	0.868	0.917	0.962	1.003	1.040	
Value of $0.456 + 0.01\ h_s$	0.806	0.856	0.906	0.956	1.006	1.056	

in theory at least, possible to derive the values of S, a and c in the model. Equation (12) for the horizontal plane, and Equation (13) for the four vertical planes, give us five simultaneous equations in three unknowns—S, a and c. These could be solved using, for example, a least squares technique.

In practice this is quite difficult. The matrix produced by the least squares technique is in general ill-conditioned—in other words small changes in the illuminance values (which may be caused by random errors in the measuring device) may give rise to very large changes in the calculated S, a and c.

There are, however, alternative approaches to this problem. One way would be to average the illuminance values over a very long period of time (e.g. a year) in the hope that this would generate more consistent values for S, a and c. A more promising way is to use the illuminance values for the horizontal cell with the guard ring as well. This would give a sixth equation in a and c alone (the guard ring removes the direct solar component S). A third way would be to constrain one of the components in some way. For example either a or c could be replaced by K_a aBERLIN or K_c cBERLIN where aBERLIN and cBERLIN are given by Equations (9) and (10).

One obvious use for the model, once suitably accurate values of S, a and c have been found for England, is to predict internal illuminances. It would not be too hard to modify existing daylight factor computer programs to calculate point-to-point internal illuminances under a sky with the luminance distribution of Equation (8) and it would be possible to use these values to help predict lighting energy savings due to photoelectric controls[1, 2].

One recently proposed design tool is the average daylight factor, which represents the average of a series of daylight factors measured at points throughout a room. Lynes[16] and Longmore[17] have both given formulae for this quantity, which can in principle be used for daylight design in its initial stages, for example in window sizing.

A similar approach for the 'average sky' consists of the calculation of average daylight illuminance. (This represents an average both over all points in the room, and over a range of sky conditions for a particular solar azimuth and altitude). This calculation can be carried out using the vertical illuminance E_v derived in Equation (13), and analysis by Lynes[16] as follows:

$$\text{Flux entering window} = E_v . T . W \quad (16)$$

where T = transmittance of window

W = window area

Let E_{in} be the average illuminance on all the room surfaces.

$$\text{Then flux striking indoor surfaces} = E_{in}.A \quad (17)$$

where A is the total area of all indoor surfaces, ceiling, floor, walls and windows.

$$\therefore \text{flux absorbed by indoor surfaces} = E_{in}.A\,(1-R) \quad (18)$$

where R = area-weighted mean reflectance of all the surfaces.

Now the flux entering the room must equal the flux absorbed.

$$\therefore E_V.T.W = E_{in}\,A\,(1-R) \quad (19)$$

which gives average indoor illuminance

$$E_{in} = \frac{W\,T\,E_v}{A\,(1-R)} \quad (20)$$

substituting for E_V equation (13), we get

$$E_{in} = \frac{W\,T}{A\,(1-R)} (S \cos h_s \cos \phi_{REL}\, G(\phi_{REL}) + \frac{\pi c}{2} + a\,J) \quad (21)$$

So once we know how S, a and c vary with solar altitude in Britain, we can calculate the average daylight illuminance in a side-lit room for any particular solar altitude and azimuth.

This method essentially multiplies the vertical illuminance on the window plane by a factor which depends on the properties of the room and not on orientation or solar altitude.

It is, however, impossible to obtain a single figure ratio corresponding to the traditional meaning of average daylight factor (calculated using an unobstructed horizontal illuminance) as this will vary with solar position.

7 Conclusion

The main conclusion of this paper is that sky luminance averaged, for several years, over a succession of real skies can be represented by three components

(1) a direct solar component, S,

(2) a uniform sky component, c, (whose magnitude is a function of solar altitude h_s) and

(3) a circumsolar component whose magnitude varies as ae^{-by} where y is the angle between the sun and the patch of sky under consideration, and a and b are functions of solar altitude alone.

Using this mathematical model as a starting point it is possible for the first time to derive accurate and relatively simple formulae for the average illuminance on both horizontal and vertical unobstructed planes, and the average (over time and space) illuminance in a side-lit room. It is also in principle possible to calculate the time-averaged illuminance at a point in a building. These illuminances will vary with sun position and orientation, so the concept of a constant daylight factor is meaningless for the average sky model proposed here. However the average sky model is essential for energy use calculations because it incorporates the full range of naturally occurring sky conditions.

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References

[1] Hunt, D. R. G., Ltg. Res. & Technol., **9**, (2), 93-102 (1977).
[2] Crisp, V. H. C., Ltg. Res. & Technol., **10**, (2), 69-82 (1978).
[3] Hunt, D. R. G., Building & Environment, 14, 21-33, (1979).
[4] Uglow, C. E., and Petherbridge, P., Build. Ser. & Environ. Eng., **3**, (6) 6-9, (1981).
[5] Loudon, A. G., BRE CPR 73 (1967).
[6] Crisp, V. H. C., and Lynes, J. A., A model of daylight availability for daylighting design, Proceedings of National Lighting Conference, Canterbury, 1980 (CIBS, London 1980).
[7] Wegner, J., Dissertation, Tech. Univ., Berlin (1975).
[8] Wegner, J., Gesundsheitsingenieur, **96**, (5), 127-131 (1975).
[9] Aydinli, S., The calculation of the available solar radiation and daylight, Proceedings of CIE Conference, Berlin (1980).
[10] Petherbridge, P., in Some fundamental data used by Building Services Engineers, IHVE (CIBS), London (1973).
[11] Vilhelm, V., in Rektorys, K., A survey of applicable mathematics, 120-124, Iliffe, London (1969).
[12] Hopkinson, R. G., J. Opt. Soc. Am., **44**, 455 (1954).
[13] Nagata, T., Trans. of Arch. Inst. Japan, (186) 41-48 (1971).
[14] Krochmann, J., and Seidl, M., Ltg. Res. & Technol., **6**, (3) 165-171 (1974).
[15] Bartholomew's Physical Atlas, Vol III—Atlas of Meteorology, Bartholomew's, Edinburgh (1899).
[16] Lynes, J. A., Ltg. Res. & Technol., **11**, (2) 102-106 (1979).
[17] Longmore, J., Lt. & Ltg., **68**, (3) 113-119 (1975).